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Analyzing the Percentiles of Run Length Distribution in Coefficient of Variation Chart for Effective Monitoring of Multivariate Processes

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ABSTRACT

Historically, the dominant performance metric used in control charts within the statistical process control area has been average run length (ARL). However, because of the skewness of the distribution of run length, relying only on ARL presents difficulties. This study examines the multivariate coefficient of variation (MCV) chart’s performance with the alternative indicator of its performance being the median run length (MRL). The main problem this study attempts to solve is how the distribution of run length is skewed, which makes ARL unreliable when used as the only performance metric. A severely skewed distribution may cause the ARL to misrepresent the control chart's actual performance, which might result in misunderstandings and ineffective process monitoring. This study's goal is to determine if, in light of ARL's drawbacks, the MRL offers a more accurate and trustworthy way to gauge the MCV chart’s performance. The method does this by calculating the percentiles or percentage points for the distribution of run length for the MCV chart through a series of numerical simulations. This approach enables a detailed analysis of the distribution's behaviour under varying conditions of coefficient of variation shifts. The primary findings indicate that when the coefficient of variation shift increases, the skewness decreases for the distribution of run length. This pattern indicates that because the MRL is less affected by the skewness compared to the ARL, it offers a more stable and reliable performance measure. Moreover, a deeper comprehension of the MCV chart's functionality and performance may be gained by analyzing the percentiles for the distribution of run length. The study concludes that when compared to the ARL, the MRL can be a more accurate and significant performance metric for the MCV chart. The MRL and percentiles for the distribution of run length make it easier to assess the chart's performance in a more thorough manner. These findings hold significant implications for improving the reliability and applicability of MCV charts across various industrial and research contexts. By adopting MRL as an alternative performance measure, practitioners can achieve a more accurate and robust assessment of process control in multivariate settings, leading to better decision-making and enhanced process quality.

Keywords: average run length, multivariate coefficient of variation chart, percentile of the run length distribution

INTRODUCTION

The average run length or ARL has been super important for designing control charts. But what does that mean? It is the average number of samples plotted on a control chart before any signals detected by the control chart (Lee and Khoo, 2006). On the other hand, using the ARL might be challenging at times. In fact, when there is a bigger shift in the process, the run length distribution might move from being abnormally skewed to practically symmetric. Many researchers have taken a good look at run length distributions in control charts. For instance, Teoh et al. (2017) pointed out that using variable sample sizes is important for finding moderate shifts in mean values. They have done a lot of studies focusing on run length criteria. Since the shape of these run length distributions changes depending on how big the mean shifts are, just looking at the ARL can get confusing. Instead, using percentiles from the run length distribution might make more sense. Teoh et al. (2016) also explored Shewhart ($\bar{X}$) charts where process parameters were estimated. They found that when the process parameters are not known, relying on ARL is not always helpful. In those cases, the shape of the distribution for run length changes according to the process shift. Moreover, Lim et al. (2019) showed that when shifts are small, we see a right-skewed run length distribution for the variable sample size coefficient of variation chart; but as shifts get bigger, it can become almost symmetric. Yeong et al. (2021) evaluated the synthetic coefficient of variation chart based on different percentiles of the run length distribution.

Basically, calculating median run length (MRL) is finding the halfway point (the 50th percentile) in terms of run lengths — and it provides a much better measure for analyzing control charts since these distributions can be skewed. The MRL has been suggested as an alternative performance criterion to design control charts, see Hu et al. (2020), Chong et al. (2022), Qiao et al. (2022), Hu et al. (2023), Kumar and Sonam (2023) and Lee et al. (2023).

For monitoring multivariate data of coefficient of variation (CV), the control chart is called the multivariate coefficient of variation (MCV) chart, introduced by Yeong et al. (2016). Many researchers have looked into the MCV charts and their effectiveness. Now here comes this study's big goal: we are diving into how to use the percentage points for the distribution of run length — especially MRL — to better evaluate how well the MCV chart performs.

The gap in current research is notable. A lot of research still depends heavily on ARL as their main performance measure despite its drawbacks. The skewness in the run length distribution makes ARL an unreliable metric in certain conditions, see Hu et al. (2020), Chong et al. (2022), Qiao et al. (2022), Hu et al. (2023), Kumar and Sonam (2023) and Lee et al. (2023).

This research highlights why MRL is more stable compared to ARL for the MCV chart and how it gives us more meaningful insight since it is not overly influenced by skewness. Note that the control charts for monitoring MCV in the literature review such as Ayyoub et al. (2020), Ayyoub et al. (2021), Adegoke et al. (2022), Ayyoub et al. (2022), Chew et al. (2022), Haq and Michael (2022), Ng et al. (2022), Chew et al. (2023) and Hu et al. (2023) were not based on MRL.

This paper’s aim is straightforward: MRL offers better performance evaluations for MCV charts which are essential for monitoring processes effectively. The outline is given as:
(a) We will check out how skewness plays into the MCV chart's run length distribution.
(b) Next up: compare performance based on MRL versus ARL.
(c) We will also dive deep into what happens with the MCV chart by looking closely at its percentage points (percentiles).
(d) Lastly, we will show some practical uses of MCV charts using MRL through examples.

By hitting these targets, we hope to confirm that MRL gives us something truly valuable and reliable for evaluating MCV charts. This way, we are boosting accuracy and effectiveness when keeping an eye on processes within multivariate environments.

Here is how this paper flows: Section 2 shares about MCV chart and we will find formulas for calculating the percentage points for the distribution of run length. In Section 3, we discuss how well the MCV chart does — including an example showcasing its application based on MRL. Finally, in Section 4, we will conclude this research.

REVIEW OF MCV CHART

Let us consider a random sample of size n with number of quality characteristics of p, then $\mathbf{X}_i \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ for $i = 1,2, \ldots, n$. Here, $\boldsymbol{\mu}$ denotes the process mean vector and $\boldsymbol{\Sigma}$ denotes the process covariance, where $\mathbf{X}_i^T = (X_{i1}, X_{i2}, \ldots, X_{ip})$. Consequently, the MCV is defined as $\gamma = \sqrt{\boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}}$ (Voinov and Nikulin, 1996).

Let $\overline{\mathbf{X}}$ is normally distributed with mean vector of $\boldsymbol{\mu}$ and the covariance matrix of $\boldsymbol{\Sigma}$, whereas $\mathbf{S}$ follows a Wishart distribution with degrees of freedom is given as $(n-1)$ and covariance matrix is given as $\boldsymbol{\Sigma}/(n-1)$. To compute the sample MCV (denoted as $\hat{\gamma}$), $\boldsymbol{\mu}$ is estimated from the sample mean vector $\overline{\mathbf{X}} = \sum_{i=1}^{n} \mathbf{X}_i / n$ and $\boldsymbol{\Sigma}$ is estimated from the sample covariance matrix $\mathbf{S} = \sum_{i=1}^{n} (\mathbf{X}_i - \overline{\mathbf{X}})(\mathbf{X}_i - \overline{\mathbf{X}})^T / (n-1)$. Consequently, according to Yeong et al. (2016), $\hat{\gamma}$ is computed as

$$\hat{\gamma} = \sqrt{\overline{\mathbf{X}}^T \mathbf{S}^{-1} \overline{\mathbf{X}}}. \tag{1}$$

Let p be the number of quality characteristic, then the cumulative distribution function of the $\hat{\gamma}$ is calculated as

$$F_{\hat{\gamma}}(x | n, p, \delta) = 1 - F_F\left(\frac{n(n-p)}{(n-1)px^2} \middle| p, n-p, \delta\right), \tag{2}$$

where $F_F(\cdot\, | p, n-p, \delta)$ is the non-central F distribution, where δ denotes the non-centrality parameter, calculated as $\delta = n\boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} = n/\gamma^2$ with the degrees of freedom are p and $(n-p)$. The inverse of the cumulative distribution function of $\hat{\gamma}$ is computed as

$$F_{\hat{\gamma}}^{-1}(x | n, p, \delta) = \sqrt{\frac{n(n-p)}{(n-1)p}\left[\frac{1}{F_F^{-1}(1-x | p, n-p, \delta)}\right]}, \tag{3}$$

where $F_F^{-1}(\cdot\, | p, n-p, \delta)$ is the inverse of the cumulative distribution function of the non-central F distribution with non-centrality parameter is calculated as $\delta = n/\gamma^2$, where the degrees of freedom are p and $(n-p)$.

For the MCV chart, let α_0 represent the likelihood of Type I error and $\delta_0 = n/\gamma_0^2$, where γ_0^2 is the known in-control MCV. Two one-sided charts – one labeled the upward MCV chart, while the other one is the downward MCV chart – are examined in this study. According to Yeong et al. (2016), $\text{ARL}_0 = 1/\alpha_0$ for both upward and downward MCV charts, where ARL_0 is the in-control average run length.

Due to a decrease in γ, the process is out-of-control when $\hat{\gamma}$ < LCL, then the lower control limit LCL is calculated as

$$\mathrm{LCL} = F_{\hat{\gamma}}^{-1}(\alpha_0|n, p, \delta_0) \quad (4)$$

for the downward MCV chart. Due to an increase in γ, the process is out-of-control when $\hat{\gamma}$ > UCL, then the upper control limit UCL is calculated as

$$\mathrm{UCL} = F_{\hat{\gamma}}^{-1}(1 - \alpha_0|n, p, \delta_0) \quad (5)$$

for the upward MCV chart. Here, $\delta_0 = n/\gamma_0^2$. Let ARL_1 be the out-of-control average run length, then $\mathrm{ARL}_1 = 1/\beta$, where β is the probability in detecting a process shift with the size of $\tau = \gamma_1/\gamma_0$, where γ_1 is the out-of-control MCV. Here,

$$\beta = F_{\hat{\gamma}}(\mathrm{LCL}|n, p, \delta_1) \quad (6)$$

for the downward MCV chart, whereas

$$\beta = 1 - F_{\hat{\gamma}}(\mathrm{UCL}|n, p, \delta_1) \quad (7)$$

for the downward MCV chart, where $\delta_1 = n/(\tau\gamma_0)^2$. Note that $0 < \tau < 1$ (i.e. $\gamma_1 < \gamma_0$) for the downward MCV chart in detecting a decreasing multivariate CV; while $\tau > 1$ (i.e. $\gamma_1 > \gamma_0$) for the upward MCV chart in detecting an increasing multivariate CV.

PERCENTAGE POINTS OFR DISTRIBUTION OF RUN LENGTH

As the number of plotted samples until the first out-of-control signal is triggered, the run length (RL) of the MCV chart is determined. The ARL and MRL, respectively, are the average and median values of the RL. The RL's probability distribution function

$$\Pr(\mathrm{RL} = l) = \beta^{l-1}(1 - \beta) \quad (8)$$

is provided by Brook and Evans (1972), where $l \in \{1, 2, 3, \ldots\}$ and the cumulative distribution function of RL is calculated as

$$\Pr(\mathrm{RL} \leq l) = 1 - \beta^l, \quad (9)$$

then the 100ρ percentage point of the RL distribution can be determined as m value such that

$$\Pr(\mathrm{RL} \leq m - 1) \leq \rho \text{ and } \Pr(\mathrm{RL} \leq m) > \rho \quad (10)$$

where $0 < \rho < 1$. By setting $\rho = 0.5$ in Eq. (10), we can observe that m = MRL, where calculating the 50th percentile for the distribution of RL is giving the value of MRL.

RUN LENGTH PERFORMANCE FOR THE MULTIVARIATE COEFFICIENT OF VARIATION CHART

Figures 1-4 and 5-8 show some graphs. They display the probability distribution of RL for both upward (Figures 1-4) and downward (Figures 5-8) MCV charts, where the probability is calculated using Eq. (8). This applies when $p = 2$, $n = 5$, $\gamma_0 = 0.5$ and $ARL_0 = 370$. The UCL computes to be 1.319976 for the upward MCV chart. Meanwhile, the LCL stands at 0.050858 for the downward MCV chart. Based on these figures, it is pretty clear that as τ increases, the skewness for the distribution of run length gets smaller. This means that judging the MCV chart's performance only by ARL is not enough. Instead, if we look at MRL for evaluating the upward and downward MCV charts, we can get more useful insights.

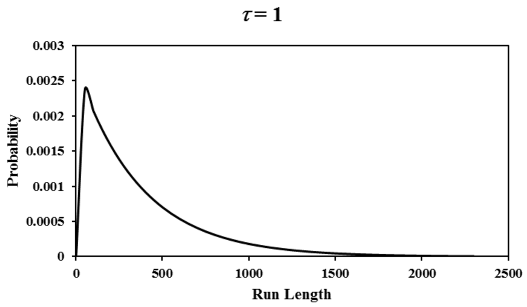

Figure 1: The distribution of RL when τ = 1.00 for upward MCV chart

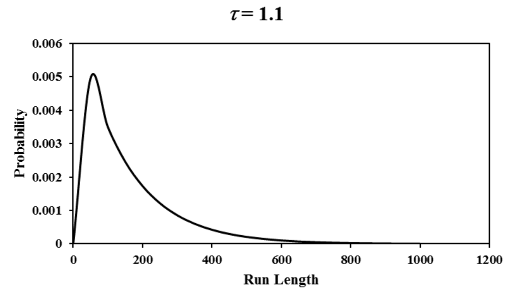

Figure 2: The distribution of RL when τ = 1.10 for upward MCV chart

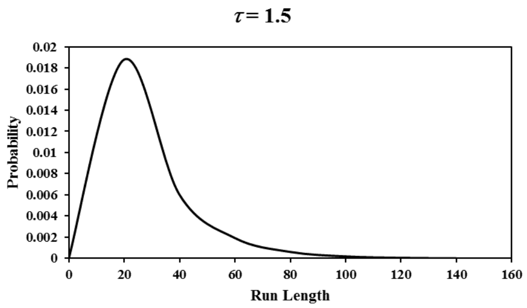

Figure 3: The distribution of RL when τ = 1.50 for upward MCV chart

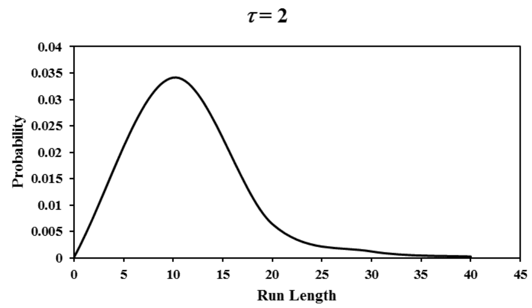

Figure 4: The distribution of RL when τ = 2.00 for upward MCV chart

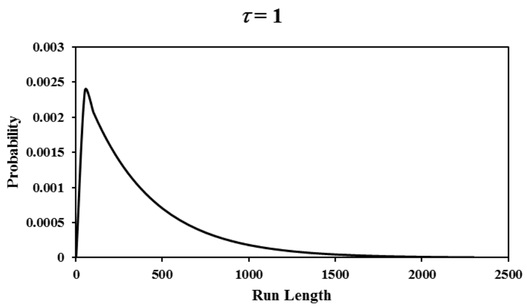

Figure 5: The distribution of RL when τ = 1.00 for downward MCV chart

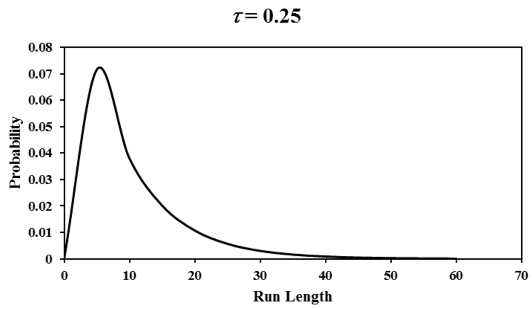

Figure 6: The distribution of RL when τ = 0.25 for downward MCV chart

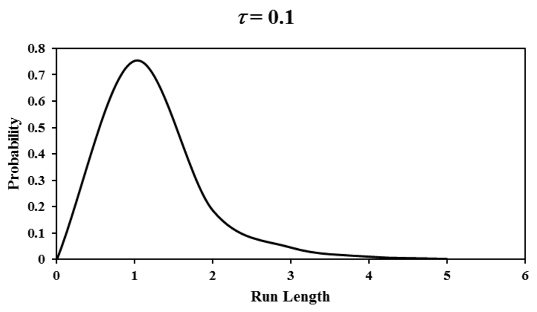

Figure 7: The distribution of RL when τ = 0.10 for downward MCV chart

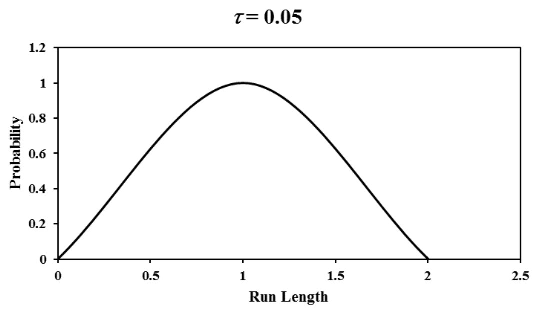

Figure 8: The distribution of RL when τ = 0.05 for downward MCV chart

Tables 1 and 2 share the UCL and LCL values based on ARL or MRL for p = 2 and $n \in \{5, 10, 15\}$, with ARL_0 or $MRL_0 \in \{250, 370, 500\}$ and $\gamma_0 \in \{0.1, 0.5\}$. Here, ARL_0 represents the in-control ARL, while MRL_0 indicates the in-control MRL. An interesting trend here is as sample size n grows larger, UCL dips down while LCL climbs up. Also noteworthy is how UCL and LCL calculated using MRL are quite different from those using ARL; specifically UCL computed from MRL is greater than the one calculated using ARL, while LCL computed from MRL ends up being less than the one calculated using ARL.

Table 1: UCL for the upward MCV chart (p = 2)

γ_0	MRL_0 or ARL_0		MRL-based UCL			ARL-based UCL		
		n =	5	10	15	5	10	15
0.1	250		0.189821	0.162846	0.151103	0.184364	0.159431	0.148441
	370		0.195542	0.166420	0.153888	0.190237	0.163106	0.151306
	500		0.199822	0.169092	0.155968	0.194626	0.165849	0.153443
0.5	250		1.313890	0.995483	0.880674	1.237604	0.960147	0.856262
	370		1.401535	1.034174	0.907017	1.319976	0.998234	0.882561
	500		1.473131	1.064328	0.927266	1.386928	1.027864	0.902748

Table 2: LCL for the downward MCV chart (p = 2)

γ_0	MRL_0 or ARL_0		MRL-based LCL			ARL-based LCL		
		n =	5	10	15	5	10	15
0.1	250		0.010952	0.035432	0.047459	0.012381	0.037308	0.049200
	370		0.009597	0.033537	0.045717	0.010849	0.035292	0.047347
	500		0.008668	0.032166	0.044430	0.009804	0.033835	0.045995
0.5	250		0.051343	0.164934	0.221844	0.058058	0.173870	0.230236
	370		0.044980	0.155878	0.213307	0.050858	0.164268	0.221215
	500		0.040645	0.149446	0.207089	0.045953	0.157351	0.214655

The percentage points for the upward (refer to Table 3) and downward (refer to Table 4) MCV charts for the distributions of run length are calculated using Eq. (10). These statistics demonstrate that ARLs are generally greater than comparable MRLs; in this instance, MRL represents the 50th percentile. When τ = 1, it can be seen via closer inspection that the value of ARL_0 lies between the 60th and 70th percentiles of distribution for run length. This implies that there is a rightward tilt in the in-control distribution of run length.

The lower percentage points in Tables 3 and 4 (e.g. the 1st, 5th, and 10th percentiles with τ at 1.00) give us a peek into early false alarms. For instance, there is a chance of about 10% that we might get a false alarm by the time we hit the 39th sample point when $\tau = 1$ for the MCV chart.

Table 3: ARLs and percentage points of the RL distribution when $p = 2$, $n = 5$, $\gamma_0 = 0.5$, UCL = 1.319976, and $ARL_0 = 370$ for the upward MCV chart

τ	ARL	Percentage points of RL distribution										
		1st	5th	10th	20th	30th	40th	50th	60th	70th	80th	90th
1.00	370.00	4	19	39	83	132	189	257	339	445	595	851
1.25	51.84	1	3	6	12	19	27	36	48	62	83	119
1.50	18.13	1	1	2	4	7	10	13	17	22	29	41
1.75	9.70	1	1	1	3	4	5	7	9	12	15	22
2.00	6.49	1	1	1	2	3	4	5	6	8	10	14

Table 4: ARLs and percentage points of the RL distribution when $p = 2$, $n = 5$, $\gamma_0 = 0.5$, UCL = 0.050858, and $ARL_0 = 370$ for the downward MCV chart

τ	ARL	Percentage points of RL distribution										
		1st	5th	10th	20th	30th	40th	50th	60th	70th	80th	90th
1.00	370.00	4	19	39	83	132	189	257	339	445	595	851
0.80	204.55	3	11	22	46	73	105	142	187	246	329	470
0.60	92.77	1	5	10	21	33	48	64	85	112	149	213
0.40	29.94	1	2	4	7	11	16	21	27	36	48	68
0.20	4.81	1	1	1	1	2	3	3	4	6	7	10

On another note — the higher percentage points from out-of-control run length distributions tell us when an out-of-control signal might likely trigger based on certain sampling points with a higher chance of happening. For example, based on Table 3 for the upward MCV chart, there is a solid chance of about 90% for an out-of-control signal to be detected by the time we reach point number 119 when there is a shift at $\tau = 1.25$.

AN ILLUSTRATIVE EXAMPLE

The operation of MCV charts with the MRL will be examined in this section, following the example provided by Yeong et al. (2016). They made use of values such as $\gamma_0 = 0.001042$, $n = 5$ and $p = 2$. For the data, see Table 5, which includes eight sample MCVs. We can now see the upward and downward MCVs curves for those sample MCVs by looking at Figures 9 and 10. The UCL = 0.0020135 and LCL = 0.00010025 control limits were used. We use an equation we refer to as Eq. (10), which is based on these limits on $MRL_0 = 370$.

Based on Figures 9 and 10, all the sample MCVs fall within those control limits. Figure 9 shows that every sample is plotted below the UCL on the upward MCV chart. Meanwhile, Figure 10 tells us that all samples are plotted above the LCL for the downward MCV chart. It means that both upward and downward MCV charts are not giving any out-of-control signals, which suggests that everything is running smoothly and in control.

Table 5: Data for the illustrative example

Sample number	1	2	3	4	5	6	7	8
Sample MCV	0.001692	0.001069	0.000872	0.00086	0.001436	0.000846	0.000615	0.000632

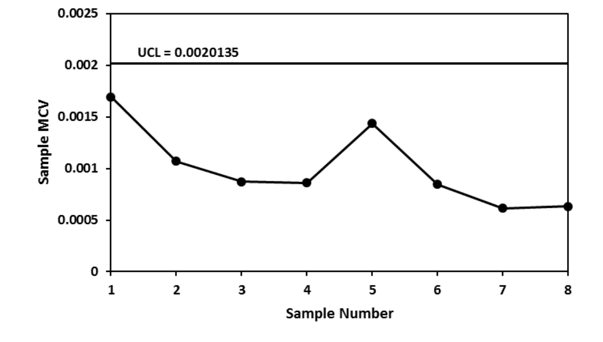

Figure 9: Illustrative example for the upward MCV chart

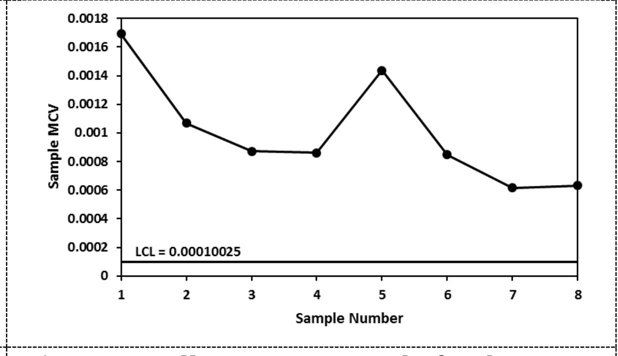

Figure 10: Illustrative example for the downward MCV chart

CONCLUSION

Based on the numerical results, it is evident that the numbers tell us something interesting. For both upward and downward MCV charts, the distribution of run length tends to the right. This skewness is variable. The distribution is skewed when there is no shift. However, the shape of the run length distribution is almost symmetry as the shift increases. We also looked at the percentage points in these charts' run length distributions. This gives us a far better grasp of the performance of the MCV charts.

For future research, we could create an MCV chart that uses adaptive schemes, that are variable sample sizes or sampling intervals. In addition, this study assumed we knew the process parameters. It might be interesting to see how the MCV chart works when we have to estimate the process parameters instead.

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