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Magnetohydrodynamics Stagnation Point Flow over a Stretching or Shrinking Sheet in a Porous Medium with Velocity Slip and Suction

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ABSTRACT
From previous studies, the current understanding is limited, and there is a lack of comprehensive research addressing the combined effects of slip, suction, porous, stretching/shrinking sheets, and magnetohydrodynamics on flow behaviour. Therefore, this study seeks to examine the magnetohydrodynamics stagnation point flow over a stretching or shrinking sheet in a porous medium in the presence of velocity slip and suction effects. By applying a suitable similarity transformation, we acquired a system of ordinary differential equations (ODEs) that are reduced from the governing system of partial differential equations (PDEs). The resulting system of differential equations subjected to the boundary conditions are solved numerically by using bvp4c in MATLAB software. The governing parameters involved in this study such as velocity and temperature profiles, skin friction coefficients and the local Nusselt number are investigated and discussed in details. The parameters that are observed are magnetic parameter M, slip parameter A, permeability parameter K, and suction parameter S towards stretching or shrinking parameter c. It is observed that there is a dual solution for the shrinking sheet ($c < 0$), while a unique solution is obtained for the stretching sheet ($c > 0$). An escalation in the magnetic parameter M, permeability parameter K, and suction parameter S contributes to an elevation in the skin friction coefficient, while an increased slip parameter A results in a reduction in the skin friction coefficient. Increasing the magnetic parameter M, permeability parameter K, slip parameter A, and suction parameter S causes an increase in Nusselt numbers.

Keywords: Magnetohydrodynamics, Porous Medium, Stagnation Point Flow, Velocity Slip

INTRODUCTION

The phenomenon of flow over a stretching sheet is prevalent in engineering applications, particularly in manufacturing. Crane (1970) pioneered the closed analytical solution for two-dimensional flow resulting from plate stretching. Subsequently, various researchers have shown interest in this topic, particularly focusing on boundary layer flow near the stagnation point on a stretching surface. Recent studies have predominantly addressed scenarios where the sheet stretches within its own plane, exhibiting a velocity proportional to the distance from the stagnation point. Examples of such investigations include Nazar et al. (2004) for micropolar fluid, Lok et al. (2006) for oblique viscous flow, and Hayat et al. (2009) for magnetohydrodynamic (MHD) flow.

Ibrahim et al. (2013) extended their studies to nanofluids whereas Saif et al. (2017) extended their studies to second grade nanofluids.

In contrast, the velocity on a shrinking sheet converges towards a fixed point, as highlighted by & Wang (2006). Flow towards a shrinking sheet can occur under two conditions: either due to significant suction on the boundary (Wang & Wang, 2006) or through the addition of a stagnation point flow (Wang, 2008). Wang (2008) introduced the similarity transformation methods in analyzing the Navier-Stokes equation by reducing the governing system of partial differential equation to a set of non-linear ordinary differential equations, which were then numerically integrated. Fan et al. (2010) delved into the dynamics of unsteady stagnation flow and heat transfer towards a shrinking sheet, noting that velocity profiles approach steady flow more rapidly than in the stretching sheet case.

Mahapatra & Nandy (2013b) conducted a stability analysis of dual solutions in stagnation point flow over a porous shrinking sheet, observing that the porosity parameter influences the existence range of similarity solutions. Sharma et al. (2014) explored the stability of magnetohydrodynamic stagnation-point flow towards a stretching or shrinking sheet, revealing the existence of dual solutions with different stability characteristics. Studies on slip effects, conducted by Mahapatra & Nandy (2013a) and Aman et al. (2013), showed that an increase in the velocity slip parameter widens the range of similarity solutions. Additionally, slip effects impact heat transfer rates at the surface. The introduction of a porous medium further influences heat transfer, as observed by Rosali et al. (2011), Vyas & Srivastava (2012), Pal & Mandal (2015), and Yasin et al. (2017). Various researchers investigated the impact of buoyancy on stagnation point flow in porous media, with findings by Hong et al. (2020) and Khan et al. (2021). Wahid et al. (2022) focused on the unsteady mixed convective stagnation point flow of a hybrid nanofluid in a porous medium, emphasising the positive influence of resistant parameters on heat transfer and skin friction rates. Whereas Swapna et al. (2024) studied the MHD stagnation point flow of an incompressible micropolar fluid on a stretching/shrinking sheet.

Hence, motivated from the previous study, we extended this study by Japili *et al*. (2022) to the problem of Magnetohydrodynamic (MHD) stagnation point flow towards a stretching or shrinking sheet in a porous medium with velocity slip by considering the S parameter, which indicates the presence of suction effects. The new effect of suction, considered in this study, have not been reported by any researchers to date. The main objective of this research is to investigate the behaviour of the flow and analyze both unique and dual solutions. Thus, by drawing on the previous research, we aim to achieve precise and accurate outcomes, particularly in demonstrating how the addition of suction affects fluid dynamics. This includes achieving consistent results that are more accurate and precise compared to previous studies that did not consider suction, thereby extending the existing research and contributing new insights to the field.

MATHEMATICAL FORMULATION

Consider a steady magnetohydrodynamics stagnation point flow of viscous incompressible fluid over a stretching or shrinking sheet in a porous medium with velocity slip and suction effects. The stretching/shrinking sheet velocity is U_w, velocity of external flow is U_∞, T_w is the surface temperature and T_∞ is the temperature at the surrounding and it is assumed to be a constant. Based on the assumptions, the governing system of partial differential equations (1) – (3) and the given boundary conditions (4) are constructed by following the previous studies (Bhattacharyya & Layek (2011); Japili *et al.* (2022)):

- continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

- momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_\infty\frac{dU_\infty}{dx}+v\frac{\partial^2 u}{\partial y^2}+\frac{v}{K_1}(U_\infty-u)+\frac{\sigma{\beta_0}^2}{\rho}(U_\infty-u), \tag{2}$$

- energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

subject to the boundary conditions,

$$u=U_w+L\frac{\partial u}{\partial y},\quad v=V_w,\quad T=T_w,\quad at\quad y=0, \tag{4}$$

$$u\to U_\infty,\quad T\to T_\infty,\quad as\ y\to\infty.$$

As shown in Figure 1, we let u and v represent the velocity components in the x and y directions, respectively. The x-axis is aligned with the stretching/shrinking sheet, while the y-axis is measured perpendicular to it. V_w is the velocity of suction and $U_\infty=ax$ is the velocity of external flow where $a>0$.

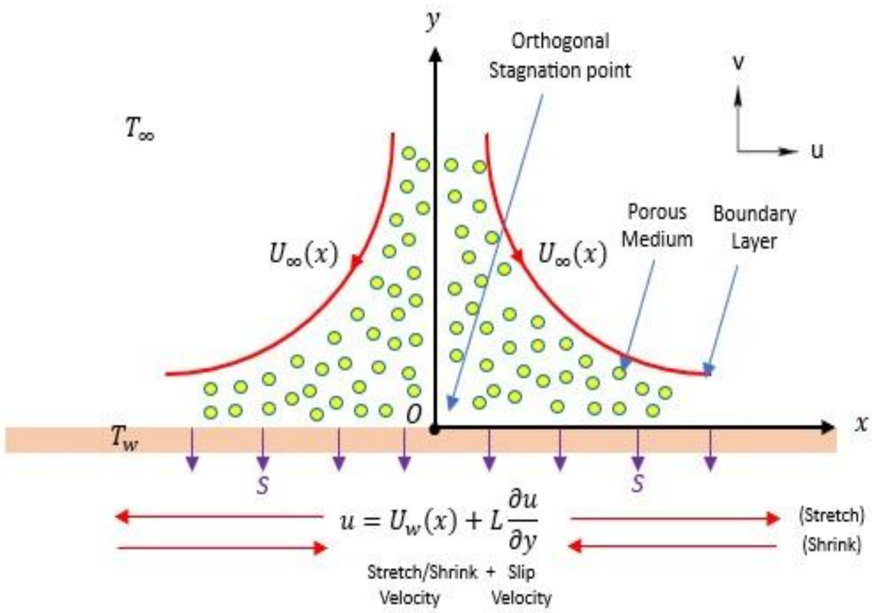

Figure 1: Schematic diagram of the problem

The governing equations (2) and (3) along with the boundary conditions (4) are converted into ordinary differential equations (ODEs), while equation (1) is satisfied. This conversion is done using the similarity transformations that were obtained from Bhattacharyya & Layek (2011) and Japili *et al*. (2022) as shown below:

$$\eta = \left(\frac{U_\infty}{\alpha x}\right)^{\frac{1}{2}} y, \quad \psi = (\alpha x U_\infty)^{\frac{1}{2}} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad V_w = -(\alpha a)^{\frac{1}{2}} S \tag{5}$$

where the similarity variable η and the function of the stream ψ that satisfied equation (1) is defined as follows:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}.$$

After applying the transformations, equations (2) and (3), along with boundary conditions (4) can be expressed as a system of differential equations (ODEs) as follows,

$$Prf''' + ff'' - (f')^2 + K(1 - f') + M^2(1 - f') + 1 = 0, \tag{6}$$

$$\theta'' + f\theta' = 0. \tag{7}$$

The boundary conditions,

$$f(0) = S, \quad f'(0) = c + Af''(0), \quad \theta(0) = 1, \tag{8}$$

$$f'(\eta) \to 1, \quad \theta(\eta) \to 0 \quad as \quad \eta \to \infty,$$

where $c > 0$ corresponds to the stretching case, and $c < 0$ correspond to the shrinking case.

The velocity slip, A is defined as $A = L(a/\alpha)^{\frac{1}{2}}$ and Pr is the Prandtl number. The permeability parameter, K and the magnetic parameter M are defined as follows,

$$K = \frac{\nu}{ak},$$
$$M = (\sigma/a\rho)^{\frac{1}{2}}\beta_0.$$

The physical quantities of interest are skin friction coefficients C_f and local Nusselt number Nu_x, are defined as:

$$C_f = \frac{\tau_w}{\rho U_\infty{}^2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)}. \tag{9}$$

where τ_w and q_w represent the surface shear stress and heat flux, respectively, defined as:

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{10}$$

By substituting (10) into (9), we derive the reduced form of the skin friction coefficients and local Nusselt number as follows:

$$C_f Re_x^{\frac{1}{2}} = Pr^{\frac{1}{2}} f''(0) \tag{11}$$

$$Nu_x Re_x^{-\frac{1}{2}} = -Pr^{\frac{1}{2}} \theta'(0)$$

where $Re_x = \frac{ax^2}{\nu}$ is the local Reynolds number and $Pe_x = \frac{U_\infty x}{\alpha}$ is the local Peclet number.

RESULTS AND DISCUSSION

The system of ordinary differential equations (6) and (7) along with the boundary conditions (8) have been solved numerically using the bvp4c in MATLAB software. The bvp4c function in MATLAB is a numerical solver for boundary value problems (BVPs). It uses a finite difference method combined with a collocation technique to approximate solutions and adjusts the solution iteratively to satisfy the boundary conditions. This research examined specific values of the governing parameters such as the magnetic parameter M , velocity slip parameter A stretching/shrinking parameter c and permeability parameter K for a constant value of Prandtl number Pr. As in the previous problems, the computation was carried out until the solution converged at the smallest value of c, where the results for the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ are both convergent.

The comparison of $f''(0)$ for the shrinking case ($c < 0$) with the result obtained by Bhattacharyya *et.al* (2011) and Japili *et al*. (2022) for various values of c is presented in Table 1. The comparison demonstrates excellent agreement.

Table 1: Comparison values of the skin friction coefficient $f''(0)$ when $Pr = 1$ and $M = K = A = 0$.

c	Bhattacharyya *et.al* (2011)	Japili *et al*. (2022)	Present Results
-1.2465	0.58429	0.58429	0.58428
-1.15	1.08223	1.08224	1.08223
-1	1.32882	1.32882	1.32882
-0.75	1.48929	1.48930	1.48930
-0.5	1.49566	1.49567	1.49567
-0.25	1.40224	1.40224	1.40224

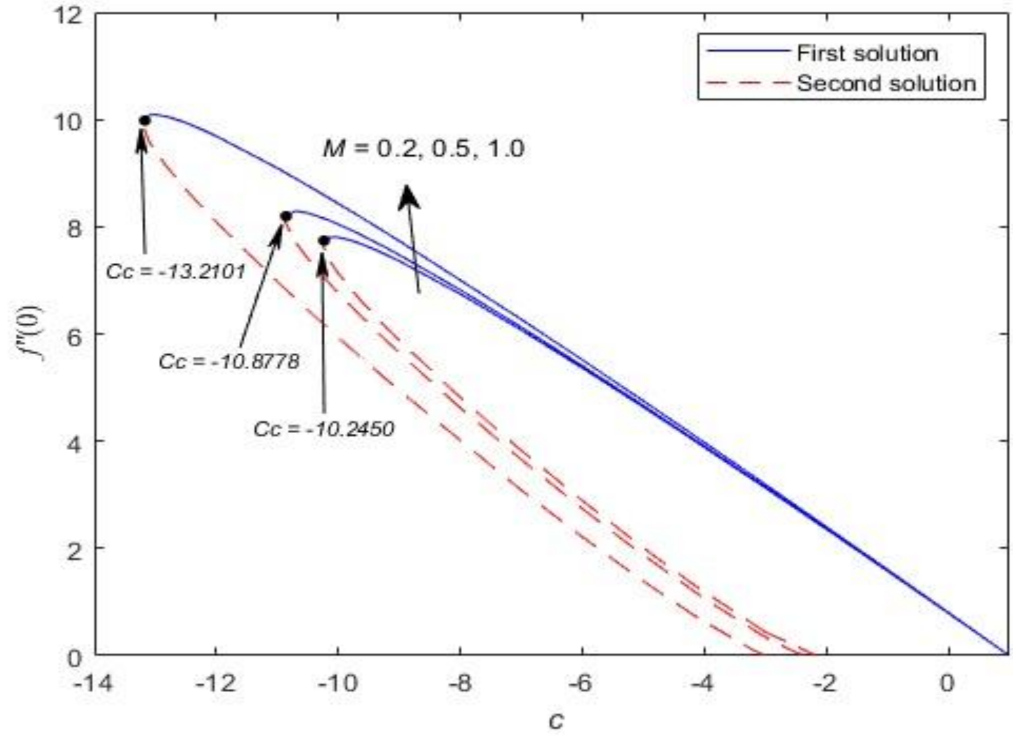

Figure 2: Effect of various M on the skin friction coefficient $f''(0)$.

Figures 2 and 3 illustrate the results of skin friction coefficients $f''(0)$ and local Nusselt number $-\theta'(0)$ for different values of M when $S = 1$. The parameters Pr, K and A remain constant. The results obtained shows that the existence of unique solution for the stretching case $(c > 0)$, while there exist dual solutions for the shrinking case $(c_c < c < 0)$. In contrast, no solution can be obtained when $c < c_c$ due to the the boundary layer separation from the surface where the boundary layer approximations are not physically possible. This clearly demonsrates that increasing the value of M influences the skin friction coefficient and heat transfer coefficient causing them to rise. The Lorentz force, or electromagnetic force opposes the fluid motion in the presence of a magnetic field. Consequently, this generates heat and delays the boundary layer separation. As the value of M increases, the delay in thermal boundary layer separation becomes more significant.

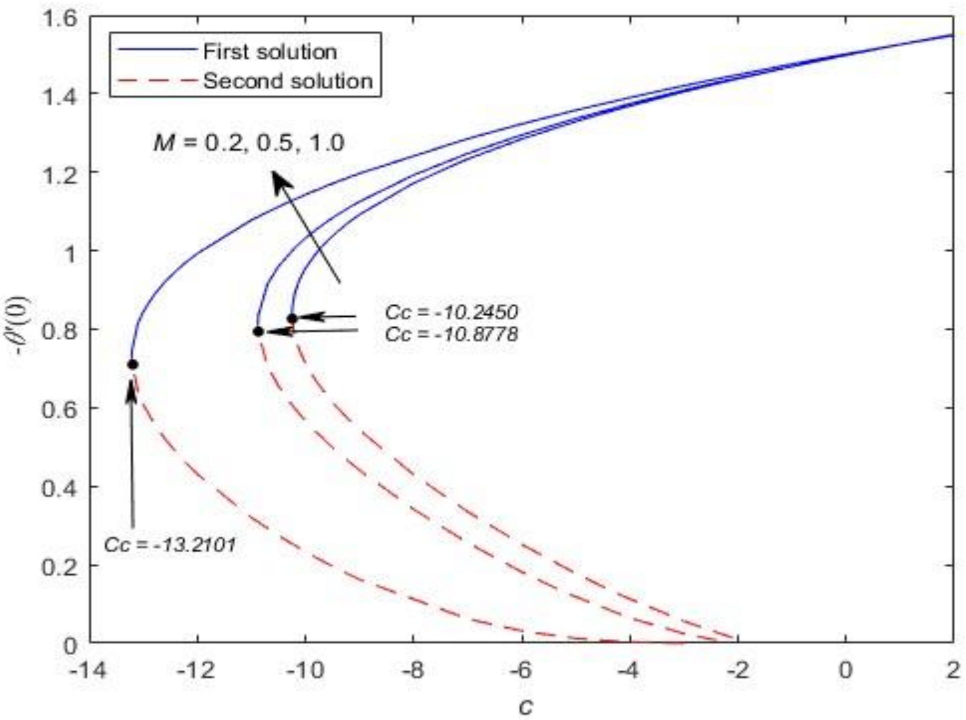

Figure 3: Effect of various M on the local Nusselt number $-\theta'(0)$.

Figures 4 and 5 display the effect of A on the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ by taking various values of A when $S = 1$, meanwhile the other parameters Pr, K and M remain constant. $A = 0$ refers to a situation where the fluid velocity at the surface of a solid boundary is equal to the velocity of the solid surface itself. In simpler terms, there is no relative motion or slippage between the fluid and the solid surface. In Figure 4, it can be seen that the value of $f''(0)$ equals zero when $c = -3$. This occurs because the fluid and the solid surface move at the same velocity. Therefore, there is no friction at the fluid-solid interface, resulting in no heat transfer between fluid and solid surface when c = −3 as illustrated in Figure 4.

From Figure 4, it is detected that the skin friction coefficient decreased when A increase from 0 to 2. This is due to the fact that when velocity slip increases, the fluid particles near the surface experience less resistance to motion. This reduced resistance resulted in a decrease in shear stress at the fluid-solid interface. However, the opposite effect is observed in Figure 5 where the heat transfer occurs at the boundary and it increased with the increasing of slip parameter A. With higher velocity slip, there is an increased relative motion between the fluid and the solid surface. This enhanced motion promotes better mixing of fluid layers and improves the advection of heat, facilitating more efficient heat transfer.

Figures 6 and 7 depict how suction influences the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ with different values of S where parameters Pr, K, M, and A kept constant. It has been observed that a dual solution is present in the case of shrinking ($c < 0$), while a unique solution exists for stretching case ($c > 0$). Observations revealed that an increase in the suction parameter S leads to a corresponding increase in the critical value c_c.

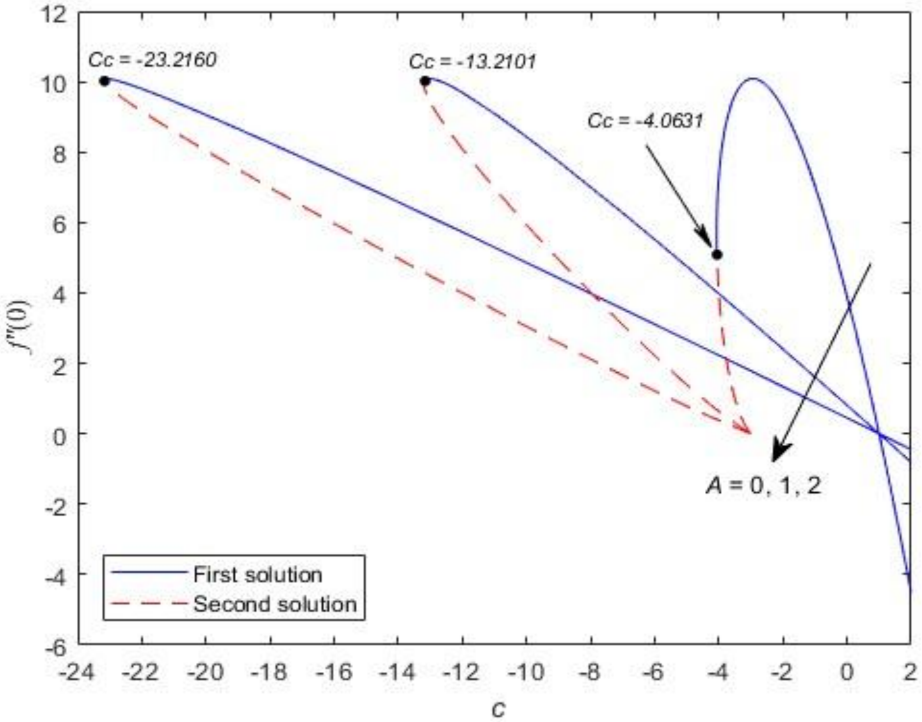

Figure 4: Effect of various A on the skin friction coefficient $f''(0)$.

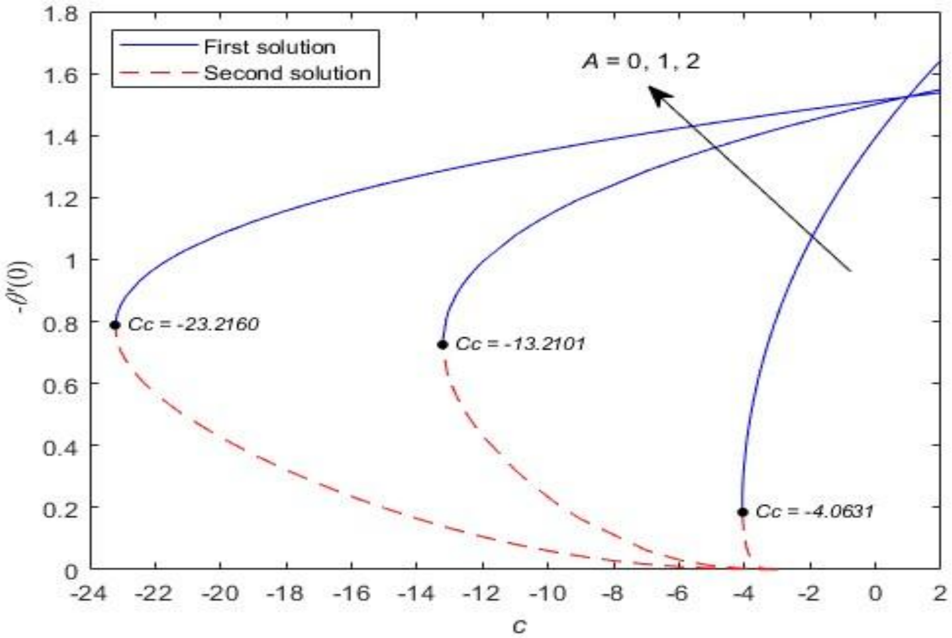

Figure 5: Effect of various A on the local Nusselt number $-\theta'(0)$.

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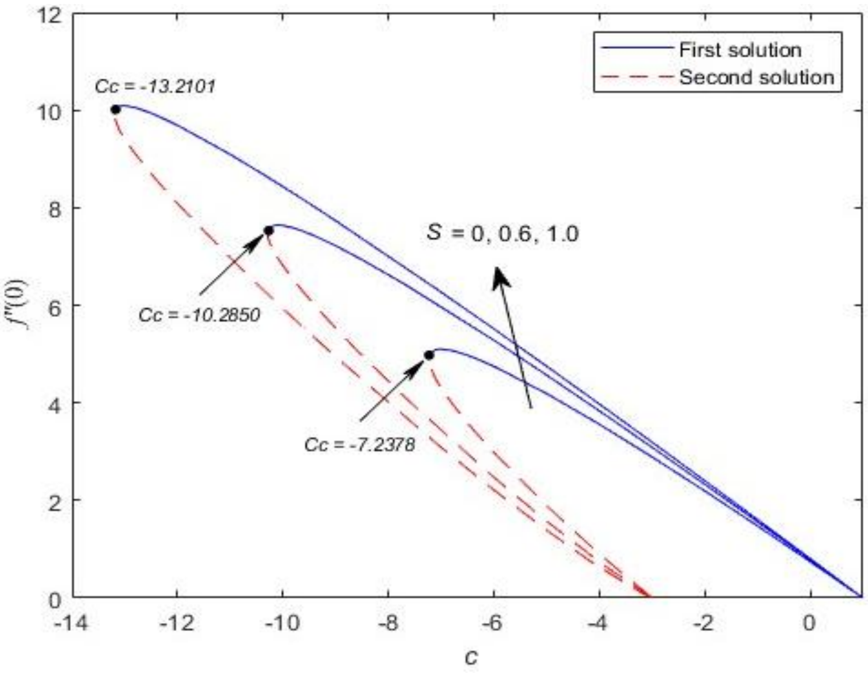

Figure 6: The skin friction coefficient $f''(0)$ for the various values of S

For Figure 6, the outcome demonstrates that with an increase in the value of S, skin friction coefficient $f''(0)$ also increase. This is due to the fact that the presence of suction creates a vacuum effect, drawing more fluid toward the surface. This increased fluid motion results in higher shear forces at the fluid-solid interface, leading to elevated skin friction. Furthermore, it has been noted that the local Nusselt number $-\theta'(0)$ also increase as the value of S increase as shown in Figure 7. Suction induces a flow of fluid toward the surface, leading to higher fluid velocities near the boundary. This increased fluid motion enhances convective heat transfer by carrying heat away from the surface more effectively.

Figures 8 and 9 display the effect of the suction parameter S on the velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles assuming other parameters remain constant. In Figure 8, it can be seen that the velocity profile increases when the parameter S increases for the first solution, whereas it decreases for the second solution. However, the opposite pattern is seen for the temperature profile, as shown in Figure 9. As parameter S increases, the temperature profile of the first solution decreases and increases for the second solution. These figures support the validity of the numerical findings obtained in this study, as the boundary conditions (8) were asymptotically satisfied.

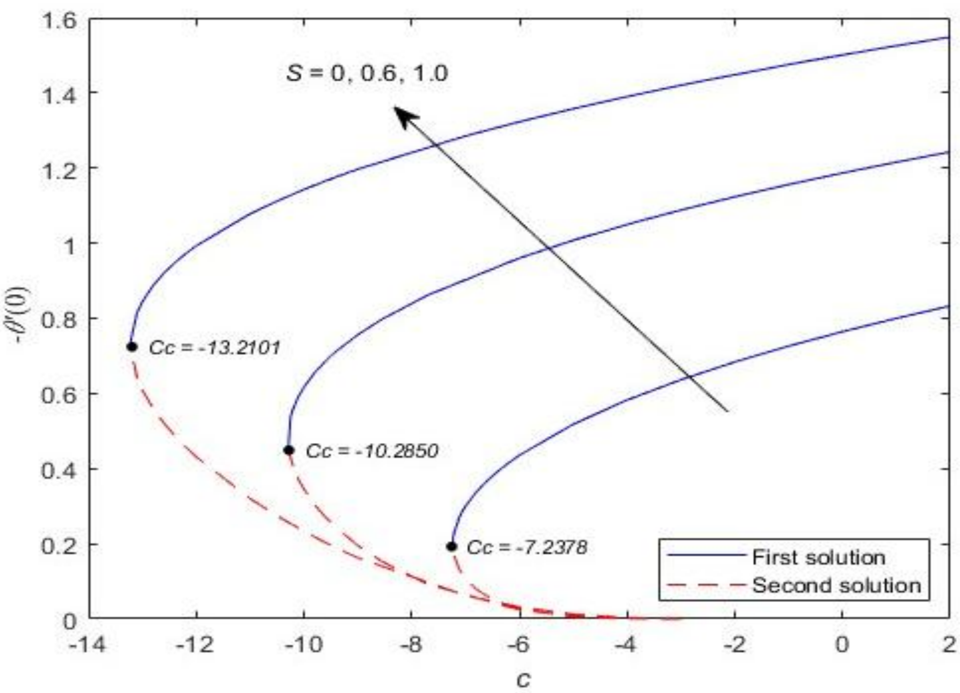

Figure 7: Effect of various S on the local Nusselt number $-\theta'(0)$.

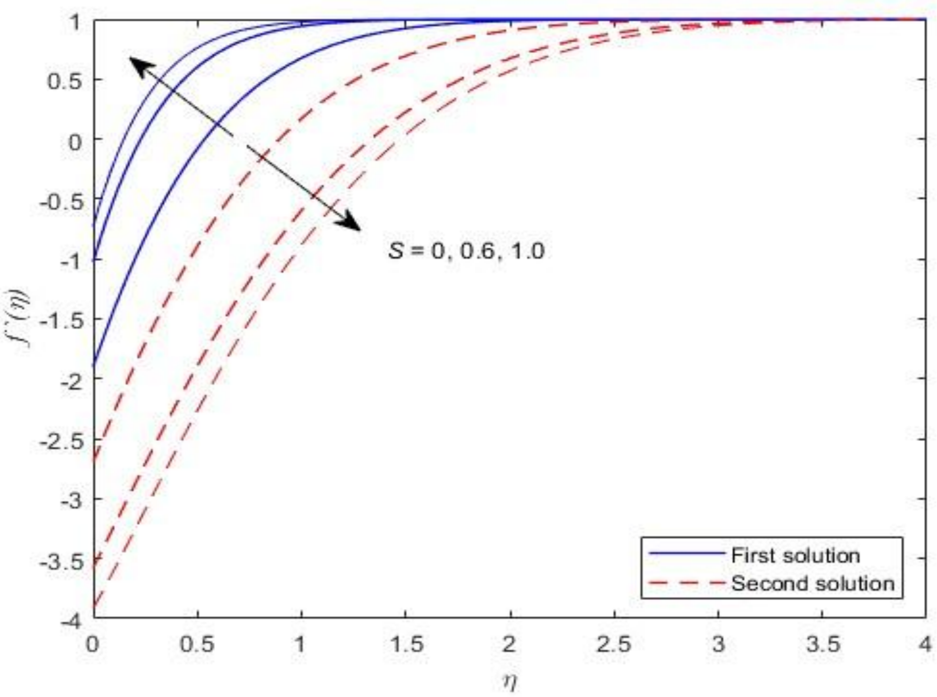

Figure 8: Effect of various S on velocity profile $f'(\eta)$.

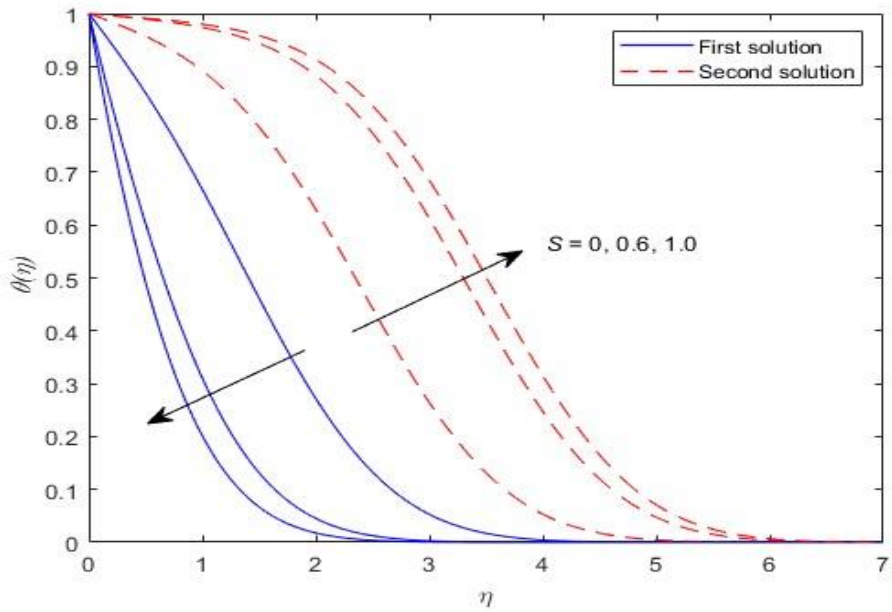

Figure 9: Effect of various S on temperature profile $\theta(\eta)$.

CONCLUSION

Magnetohydrodynamics (MHD) stagnation point flow over a stretching or shrinking sheet in a porous medium in the presence of velocity slip and suction effects has been studied accordingly. Partial differential equations (PDEs) were transformed into ordinary differential equations (ODEs) using similarity transformation. Before employing MATLAB's bvp4c solver to present the results graphically, the ODEs were numerically solved by providing an initial guess. Consequently, the following findings emerged from the research:

- Dual solutions exist for the shrinking case ($c < 0$), while for the stretching case ($c > 0$), the solution is unique.
- Velocity profiles increase with the increasing magnetic parameter M, velocity slip parameter A, permeability parameter K, and suction parameter S for the first solution, while they decrease for the second solution.
- Temperature profiles decrease with the increasing magnetic parameter M, velocity slip parameter A, permeability parameter K, and suction parameter S for the first solution, while they increase for the second solution.
- The skin friction coefficient rises in response to increased magnetic M, permeability K, and suction parameter S but decreases with a higher slip parameter A.
- Elevating the magnetic M, permeability K, slip A, and suction parameter S leads to an increase in Nusselt numbers.
- The greater the value of S, the greater the delay in thermal boundary layer separation.

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