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Heat Transfer Characteristics of Boundary Layer Flow on a Non-Linear Porous Shrinking Sheet with Radiation Effect

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ABSTRACT

This present study investigates the problem of magnetohydrodynamics (MHD) boundary layer flow over a non-linear porous shrinking sheet. This study is also considered the effect of radiation. Firstly, the governing partial differential equations are reduced to ordinary differential equations by similarity transformations. Then, the transformed equations are been solved by using bvp4c method in Matlab solver in order to obtain numerical solutions for various parameters. Parameters involved in this problem are magnetic parameter, non-dimensional controlling parameter, suction parameter, radiation parameter and Prandtl number. The present results earned are being compared with previous published results and it gives a significant agreement between them. It is found that there exist dual solutions. It is also found that the magnetic parameter delays the thermal boundary layer separation, while the non-dimensional controlling parameter enhances the thermal boundary layer separation. It is also observed that the turning point for various values of radiation parameter occurs at the same value. The impact of radiation on the heat transfer rate at the surface becomes lesser when the radiation parameter increases.

Keywords: Magnetohydrodynamics, Non-linear Porous, Radiation effect, Shrinking sheet

INTRODUCTION

The study of the dynamics of electrically conducting fluids, such as plasmas, liquid metals, and salt water or electrolytes, is known as magnetohydrodynamics (MHD). The main idea of MHD theory is the ability of conducting fluids to support magnetic fields. Magnetic fields generate forces that can alter the topology and geometry of the magnetic fields by impacting the fluid. Magnetohydrodynamics effect in chemical and manufacturing processes have received great attention among researchers. Junoh et al. (2018) mentioned that the two-dimensional steady flow of an incompressible viscous fluid in a linearly stretching plate is pioneered by Crane (1970), where Crane (1970) obtained an exact solution in closed analytical form. Later, Chiam (1994) stated that the two-dimensional and asymmetric stagnation point flows were early discovered by Hiemenz and Homann. Ali et al. (2011) reported that Hiemenz obtained an exact solution of the steady two-dimensional stagnation point flow towards a solid surface.

Radiation effects in magnetohydrodynamics (MHD) refer to the transfer of heat through the emission and absorption of electromagnetic radiation by a fluid under the influence of a magnetic field. These effects can have a significant impact on the behavior of the fluid flow and the heat transfer rate. One example of the effect of radiation on MHD flow over a stretching sheet can be found in a study by Raptis et al. (2004). Raptis et al. (2004) claimed that radiation effect on convective heat transfer and MHD flow problem becomes important in electrical power generation, astrophysical flows, solar power technology, space vehicle re-entry and other industrial areas. Damseh (2006) studied the magnetohydrodynamics mixed convection from radiate vertical isothermal surface embedded in a saturated porous media. Later, Salem (2006) considered the coupled heat and mass transfer in Darcy-Forchheimer mixed convection from a vertical flat plate embedded in a fluid saturated porous medium under the effects of radiation and viscous dissipation, and solved numerically using a shooting technique with a fourth-order Runge-Kutta integration scheme. The radiation effect on MHD mixed convection flow over a permeable vertical plate solved by Aydin and Kaya (2008). Further, Siddiq et al. (2018) solved numerically the problem of thermally and solutally convective radiation in MHD stagnation point flow of micropolar nanofluid over a shrinking sheet using RKF45 method. Other papers related to MHD and radiation effect can be found in (Babu et al., 2014; Adnan et al., 2019; Singh et al., 2020: Mahabaleshwar et al., 2022; Maranna et al., 2022).

Not many published papers on the problem of MHD together with radiation effect found in literature. Therefore, this present paper aims to study the heat transfer characteristics of MHD boundary layer flow over a non-linear porous shrinking sheet with the effect of radiation. It is worth mentioning that this present work is an extension from Nadeem and Hussain (2009) and Ali et al. (2013).

MATHEMATICAL FORMULATION

Consider a steady two-dimensional flow of an incompressible electrically conducting fluids over a nonlinear porous shrinking sheet with magnetic field. The basic governing equations are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\sigma\frac{B^2(x)}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}\,. \tag{3}$$

The boundary conditions which subject to Eqs. (1) - (3) are

$$\begin{gathered} u=-cx^n\,, v=-V_0 x^{\frac{n-1}{2}}\,, T=T_w \; at\; y=0 \\ u=0\;\,, T=T_\infty \; as\; y\to\infty \end{gathered} \tag{4}$$

where u and v are the velocity components along the x and y axes, respectively. Here c is the stretching origin, υ is the kinematic viscosity, σ is the fluid electricity conductivity, ρ is the fluid density, T is the temperature, T_w is the temperature of the surface, T_∞ is the temperature far from the surface, α is the thermal diffusivity, q_r is the radiative heat flux, C_p is the specific heat of the fluid at constant pressure α is the thermal diffusivity, $B(x)$ is the magnetic field applied normal to the shrinking sheet and porous sheet and V_0 is the porosity of the sheet. The magnetic field $B(x)$ can be written as:

$$B(x)=B_0 x^{\frac{n-1}{2}} \tag{5}$$

where B_0 is the magnetic constant. The induced magnetic field is negligible, as the magnetic Reynolds number is very small.

According to Raptis et al. (2004), the radiative heat flux, q_r can be simplified as:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$

where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. Due to the assumption that the difference of the temperature within the flow is sufficiently small, T^4 can be expressed as a linear function of temperature T using a truncated Taylor series about the free stream temperature T_∞. Thus, $T^4 \approx 4T_\infty^3 T - 3T_\infty^4$. Now, Eq. (3) reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{16\sigma^* T_\infty^3}{3k^*\rho C_p}\frac{\partial^2 T}{\partial y^2}\,. \tag{7}$$

In this study, nonlinear partial differential Eqs. (1), (2) and (7) are reduced to nonlinear ordinary differential equations using similarity transformation. The similarity transformations are as follows:

$$\begin{aligned} u &= cx^n f'(\eta) \\ v &= -\sqrt{\frac{cv(n+1)}{2}}\cdot x^{\frac{n-1}{2}}\left[f(\eta) + \frac{n-1}{n+1}\cdot\eta f'(\eta)\right] \\ \eta &= \sqrt{\frac{c(n+1)}{2v}}\cdot x^{\frac{n-1}{2}} y\,, \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \end{aligned} \tag{8}$$

where η is the similarity variables and θ is the non-dimensional temperature. We apply Eq. (8) to Eqs. (1), (2) and (7) to achieve the following ordinary differential equations:

$$f''' + ff'' - \beta f'^2 - M f' = 0 \tag{9}$$

$$\left(\frac{1}{Pr} + \frac{4}{3}R_d\right)\theta'' - f\theta' - \left(\frac{n-1}{n+1}\right)2\eta f'\theta' = 0. \tag{10}$$

Appling the similarity transformation (8) into (4), we obtain the new boundary conditions as follows:

$$\begin{aligned} &f(0) = S,\ \ f'(0) = -1,\ \ \theta(0) = 1 \\ &f'(\infty) \to 0\,, \theta(\infty) \to 0\,. \end{aligned} \tag{11}$$

The parameters that contribute in getting the results are the non-dimensional controlling parameter $\beta = \frac{2n}{n+1}$, the magnetic parameter, $M = \frac{2\sigma B_0^2}{\rho c(n+1)}$, the suction parameter, $S = -\frac{V_0}{\sqrt{\frac{cv(n+1)}{2}}}$, and the Prandtl number $Pr = \frac{v}{a}$.

The physical quantities used in the present study are the skin friction coefficient and Nusselt number given by

$$C_f = \frac{2\tau_w}{\rho u_w^2}, \qquad Nu_x = \frac{x q_w}{k(T_w - T_\infty)} \tag{12}$$

where $\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is the wall shear stress along the surface and $q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$ is the surface heat flux, with μ is the dynamic viscosity and k is the thermal conductivity. Using Eqs. (8) and (12), we obtain

$$C_f\, Re_x^{\frac{1}{2}} = [2(n+1)]^{\frac{1}{2}} \cdot f''(0), \tag{13}$$

$$Re_x^{-1/2} Nu_x = \sqrt{\frac{n+1}{2}} \cdot -\theta'(0) \tag{14}$$

where Re_x is the local Reynolds number.

RESULTS AND DISCUSSION

The Matlab bvp4c function is utilized to solve the systems of differential Eqs. (9) and (10) along with the corresponding boundary conditions (11) in a numerical manner. The solving process starts with an initial guess at the first mesh point and adapt the step size to reach the desired accuracy.

Table 1 displays the present numerical results obtained are compared to those of previous studies by Nadeem and Hussain (2009), Ali et al. (2013) and Junoh et al. (2018). This comparison reveals that the present results are in good agreement with the previous studies. Therefore, the numerical results are correct.

The numerical results from the Matlab are being collected and investigated on the relationship of the MHD and the radiation problem with the present of various parameters. The results showed that there exist dual solutions on the graph of the temperature profiles as well as the the local Nusselt number from the parameters involved. Results are validated from their profiles that asymptotically satisfying the far field boundary conditions (11).

Figures 1-4 exhibit the effect of the non-dimensional controlling parameter β, magnetic parameter *M*, Prandtl number *Pr* and radiation parameter R_d, respectively, on the temperature profiles $\theta(\eta)$. Figure 1 shows an increasing behavior for both first and second solutions of the temperature profiles $\theta(\eta)$ as β increases. The thermal boundary layer thickness for the second solution is thicker than the first solution. As for that, the dual profiles are found for this problem. Therefore, there exist dual solutions in this problem.

Further, Figure 2 displays the effect of the magnetic parameter *M* on the temperature profiles $\theta(\eta)$ when other parameters are fixed at $S = 2.4$, $\beta = 1.5$ and $R_d = 0.5$. The temperature profiles $\theta(\eta)$ show a decreasing behaviour for the first and the second solutions as the values of *M* increases. This behavior is due to the fact that the magnetic parameter generates the Lorentz force, which slows the fluid flow, and hence reduces the thickness of the thermal boundary layer. Dual profiles are also observed in Figure 2.

Table 1: Comparison values of the skin friction coefficient $f''(0)$ with different values of β when S = 1, M = 2, R_d = 0 and Pr = 6.2

β	Nadeem & Hussai (2009)	Ali et al (2013)	Junoh et al. (2018)	Present
0.0	1.86201	1.86201	1.86201	1.86200553
0.1	1.83942	1.83942	1.83942	1.839417246
0.2	1.81648	1.81648	1.81648	1.816479151
0.3	1.79318	1.79318	1.79318	1.793174504
0.4	1.76949	1.76949	1.76949	1.769485176
0.5	1.74539	1.74539	1.74539	1.745391498
0.6	1.72087	1.72087	1.72087	1.720872062
0.7	1.69591	1.69591	1.69591	1.69590349
0.8	1.67046	1.67046	1.67046	1.67046019
0.9	1.64451	1.64452	1.64452	1.644514031
1.0	1.61804	1.61804	1.61804	1.618033989

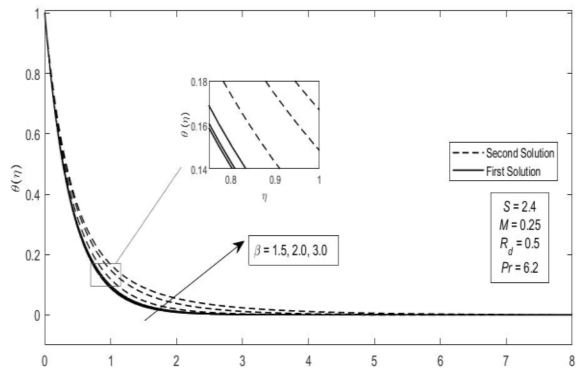

Figure 1: Temperatures profiles for various values of β

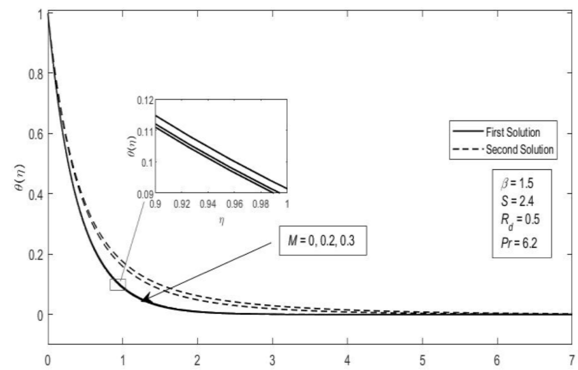

Figure 2: Temperatures profiles for various values of M

The influence of the Prandtl number, Pr on the temperature profiles $\theta(\eta)$ when S = 2.4, M = 0.25, β = 1.5 and R_d = 0.5 can be found in Figure 3. The temperature profiles $\theta(\eta)$ for both solutions show a decreasing phenomenon as Pr increases. It is also observed that the thermal boundary

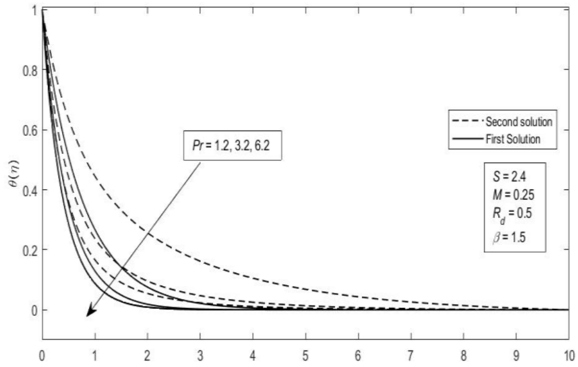

Figure 3: Temperatures profiles for various values of *Pr*

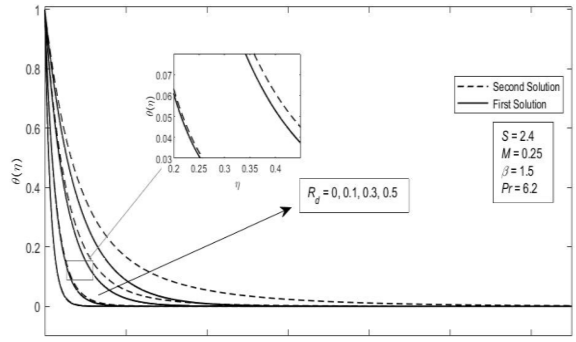

Figure 4: Temperatures profiles for various values of R_d

layer becomes thinner with *Pr*. Physically, the thermal diffusivity decreases as *Pr* increases, and these phenomena decreasing the energy ability, thus reduces the thermal boundary layer.

Figure 4 shows the temperature profiles $\theta(\eta)$ for various values of thermal radiation parameter R_d when other parameters are fixed. An increasing trend for the temperature profiles $\theta(\eta)$ discovered for both solutions when the values of R_d increases. There is an effect towards the temperature profiles of the MHD flows when the thermal radiation parameter is applied.

In this study, the results of the heat transfer rate at the surface or also known as the local Nusselt number, $-\theta'(0)$ are also obtained, as displayed in Figures 5 - 7. From Figure 5, it shows that the local Nusselt number, $-\theta'(0)$ decreases alongside with the suction parameter *S*. The same phenomenon also observed when the value of the non-dimensional controlling parameter becomes larger. For Figure 5, when β increases, the critical values of the suction parameter *Sc* decreases. Thus, we can conclude that β enhances the thermal boundary layer separation.

Figure 6 shows the local Nusselt number $-\theta'(0)$ against the suction parameter *S* for different values of the magnetic parameter *M*. The critical value of the suction paramater *Sc* is found to increase with *M*. The magnetic parameter slows the separation of the thermal boundary layer from the surface. This phenomenon is due to the fact that the magnetic parameter creates the Lorentz force which decelerates the fluid flow.

Figure 7 shows the local Nusselt number, $-\theta'(0)$ against the suction parameter *S* for various values of the radiation parameter R_d. The local Nusselt number $-\theta'(0)$ reduces alongside with the suction parameter *S.* The increasing value of the radiation parameter R_d leads to an increase in $-\theta'(0)$. It can be concluded that the change of thermal radiation affect the local Nusselt number, as in Eq. (10). The turning point *Sc* for all values of R_d occur at the same point, namely *Sc* = 1.732. When R_d = 0.5, the local Nusselt number still increases, however the increment is less compared to R_d = 0.1 and 0.3. Therefore, the impact of radiation become lesser as R_d increases.

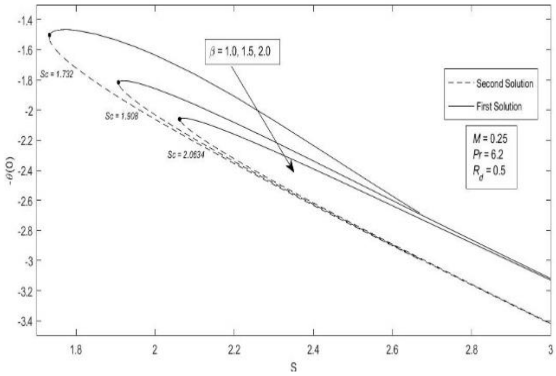

Figure 5: Variation of $-\theta'(0)$ against S for different values of β

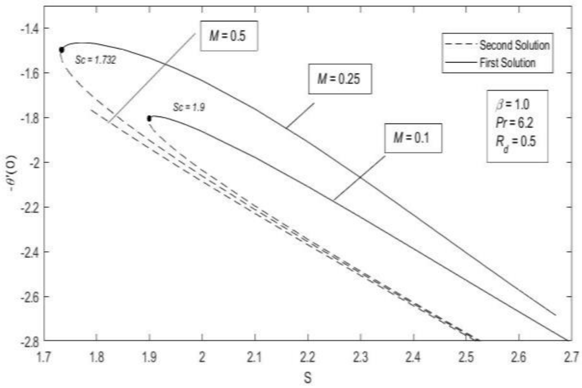

Figure 6: Variation of $-\theta'(0)$ against S for different values of M

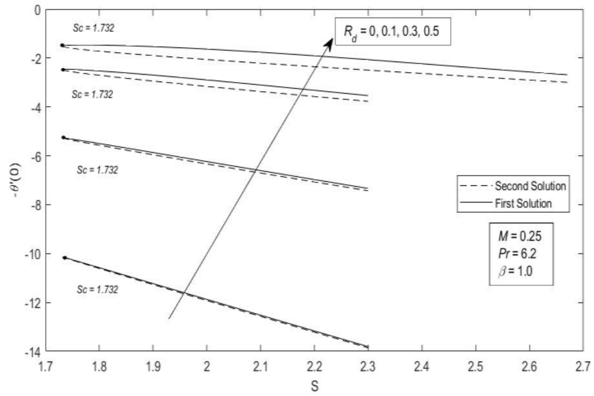

Figure 7: Variation of $-\theta'(0)$ against S for different values of R_d

CONCLUSION

The heat transfer characteristics of MHD nonlinear porous shrinking sheet is investigated with the effect of radiation. The bvp4c method in Matlab solver is used to obtain the numerical results. The present results are compared with previous published studies and it gives a favourable agreement. The main results are summarized as follows:

- The present study shows that dual solutions exist.
- For the second solution, the thermal boundary layer thickness is thicker than the first solution.
- Magnetic parameter delays the thermal boundary layer separation, however the non-dimensional controlling parameter enhances the thermal boundary layer separation from the surface.
- The thermal boundary layer separation occurs at the same turning point for various values of radiation parameter.

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