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Predicting Value-at-Risk of Bitcoin and Ethereum Using Extreme Value Theory

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ABSTRACT

Extreme value theory (EVT) has been widely used in finance especially when extreme events such as crashes, brakes and peaks occur. EVT focuses on extreme events or situations, which are typically referred to as outliers, and is able to provide better estimation for risk models. Cryptocurrency is a popular but high-risk investment due to its high volatility and occurrence of extreme events. It is difficult and critical to predict the return of cryptocurrency, mainly because of its extreme nature. This research employs two different EVT approaches, namely, block maxima approach and peaks over threshold approach, are used to model the daily extreme returns of Bitcoin and Ethereum by encrypting the left tail of cryptocurrency return distributions. Apart from that, Value-at- Risk (VaR) plays an important role in estimating the investment risk in the financial sector. Therefore, before investing, it is very important to assess the risk of cryptocurrency. In this study, VaR is evaluated by using the age-weighted historical simulation method and normal distribution. This study finds that generalized extreme value distribution using the block maxima approach fits the cryptocurrencies returns data better and normal distribution outperformed other distributions in estimating VaR. In addition, the return levels of Bitcoin and Ethereum indicates that the biggest potential losses that Ethereum will face in the next 100 years is 105.02% higher than that of Bitcoin. The result of VaR estimation also shows that the risk of Ethereum is higher than that of Bitcoin, because its high risk values are 7.03% and 8.46% respectively. The findings in this study can assist investors in understanding the behaviour of the tails in the cryptocurrency market and in making financial decisions.

Keywords Cryptocurrency; Extreme value theory; Value-at-risk; Backtesting procedure

INTRODUCTION

Throughout history, almost all transactions between human beings have been completed with money. From beans, shells, pearls, cocoa seeds, tea, pepper, animals, silver, gold and even slaves, barter finally gave way to the use of legal tender (Jenks, 1964). However, in the world of digital economy, cryptocurrency has started replacing fiat money in facilitating transactions. In this era of globalization, the development of modern technology has brought many impacts to people all over the world. The digitalization of the economy is beneficial to economic growth, and makes economic operations more effective and adaptable. For example, the introduction of electronic money or cryptocurrency such as Bitcoin, Ethereum, Dogecoin, Litecoin and many others have been used widely in performing transactions as cryptocurrencies work through Blockchain technology to enable quick transfers without any transaction fees needed.

Blockchain technology plays an important role in the cryptocurrency system. It was introduced to the public as part of the Bitcoin proposal by Satoshi Nakamoto in October 2008. A blockchain is a distributed database or ledger shared between computer network nodes, which electronically stores information in a digital form. It is also an accessible, digital and decentralised public database of bitcoin transactions, keeping permanent and verifiable record of all transactions between two parties and constantly updates digital records. Over the past few years, cryptocurrencies such as Bitcoin, Ethereum and Ripple have become more and more popular, especially in the field of financial technology. This is because cryptocurrency operates through algorithms and peer-to-peer mechanisms, and opposes the current opaque monetary system by achieving transparency and security, which is another way to change the traditional financial system (Samuelson, 1968). Apart from that, Glaser et al. (2014) it is also mentioned that the popularity of cryptocurrency has increased due to the economic crisis that caused the public to lose confidence in the financial system. In addition, Gupta (2017) pointed out that the inefficiencies and transaction costs of conventional banks contributed to the invention of bitcoin.

Due to the vigorous development of cryptocurrency industry, investors and researchers started to forecast the daily return of cryptocurrency. Many methods are used to predict the daily income. However, in this study, extreme value theory (EVT) will be used to predict the maximum extreme return, that is, the potential maximum daily losses of Bitcoin and Ethereum. This is because the extreme behaviour of cryptocurrency price fluctuation is the source of inspiration for this research. In addition, in most cases, financial data tends to follow a left-skewed or right-skewed distribution. In Parkinson's study, he pointed out that the tail of the market has important information, which can be used to analyse extreme fluctuations. Because this study emphasized more extreme data; EVT technique is used to describe extreme features (Parkinson, 1980). EVT is widely applied in many fields, ranging from hydrology to insurance and finance. It provides a powerful framework for analysing the behaviour of extreme data and pays special attention to the tails of the sample distribution. For example, to determine the likelihood of a market crash, fat tails are used. Because of its potential to predict economic situation, EVT might provide useful information to financial institutions. Therefore, compared with other methods, it is better in predicting unexpected extreme changes and daily returns.

Since cryptocurrencies are known to have high volatility, large shocks, and extreme price jumps, forecasting volatility accurately and Value-at-Risk (VaR) are important approaches for investors, professionals, and policymakers to make knowledgeable decisions and to manage portfolio risks. The objective of VaR is to recognise and understand the risk exposure, measure the risk, and manage the risk using knowledge. It even indicates the worst-case loss scenario for a certain time horizon at a specified confidence level. In addition, VaR is typically used by financial institutions for internal risk management with a one-day horizon and a 95% confidence level. The estimation of VaR through traditional non-parametric and parametric techniques is effective if there are lots of observations in the empirical distribution. Therefore, this study adopts a nonparametric method, that is, using age-weighted historical simulation, and a parametric method, that is, using the normal distribution to estimate the VaR of Bitcoin and Ethereum, two high-risk investments. From banking and health care to shipping and supply chains, many industries have been hailed as potential game changers by the blockchain technology that promotes bitcoin and other cryptocurrencies. Distributed ledgers realizes a new type of economic activity that was unrealistic before by eliminating the intermediary and middlemen in computer networks. For those who think that digital currencies has a bright future, this potential attracts them to invest in cryptocurrencies.

Although it is challenging to eliminate market risk, there are several solutions to reduce market risks. There are other techniques that eventually lead to a number called VaR, such as historical VaR technique, the Monte Carlo VaR method, generalized autoregressive conditional heteroscedasticity (GARCH) model and exponential weighted moving average (EWMA) method. Since different methods are suitable for different types of situations, it is advantageous to have a series of choices. However, if a different method are applied, it will produce different results for the same portfolio because it might lead to overestimation or underestimation events. Therefore, it is very important to evaluate the performance of VaR and select the appropriate VaR technology to provide investors with the most accurate risk level they would accept. In short, investors should predict the returns or losses of cryptocurrency before starting to invest, and understand the risks they will face, because cryptocurrency is unstable, because unexpected changes in market sentiment may lead to sharp and sudden price fluctuations.

The main objective of this study is to predict the extreme returns and analyse the risk measure of Bitcoin and Ethereum. There are three specific objectives for this study, as follows: 1. To model the cryptocurrencies' returns using block maxima approach and peak over threshold approach; 2. To estimate the parameters and examine the accuracy of Generalized Extreme Value Distribution and Generalized Pareto Distribution in predicting cryptocurrencies' returns, and 3. To investigate the relative predictive performance of Value-at-Risk approaches, age-weighted historical simulation and normal distribution.

According to CoinMarketCap, there are 20,222 different cryptocurrencies in the market, with a total market value of 919 billion US dollars. The top ten most popular cryptocurrencies are Bitcoin (BTC), Ethereum (ETH), Tether (USDT), USD Coin (USDC), BNB (BNB), Binance USD (BUSD) XRP (XRP), Cardano (ADA), Solana (SOL) and Dogecoin (DOGE). However, the main scope for this study will include only the first and second cryptocurrencies, BTC and ETH. Bitcoin was one of the earliest cryptocurrencies launched on the market in 2009, and it is also the most well-known cryptocurrency for the public. Bitcoin's market capitalization was valued around $ 391 billion with a price of $ 20,670.28. Meanwhile, another cryptocurrency, Ethereum, which adopts Ethereum blockchain technology was launched in 2015. It is the largest and longest-running open-ended decentralised software platform. Eventually, Ethereum became a platform for running secure applications using smart contracts instead of just being cryptocurrency. The market value of Ethereum is about $ 142 billion, and the price is $ 1193.49. The daily closing price data used in this study are extracted from the websites, CoinMarketCap and Yahoo! Finance from 1st January 2020 to 31st December 2022.

The remainder of the article is organized as follows. In next section, we briefly review the relative literature. The data and methods used are introduced in third section, followed by the findings of this research. Finally, we will present the conclusion and suggestions for future research.

LITERATURE REVIEW

Financial markets are crucial to capitalist economies, because they allow individuals, companies and governments to transfer funds (Mishkin, 2012). They also stabilize and regulate the circulation of money, ensuring economic stability. Financial markets make securities products profitable for investors or lenders, and provide funds for borrowers in need. In direct financial markets, lenders use assets such as stocks, futures, exchange traded funds (ETFs), bonds and mutual funds to transfer funds directly to borrowers. A financial intermediaries help transfer money between surplus and deficit accounts in the indirect market. The main task of financial intermediaries is the management of collection and credit risk. A well-functioning financial market may promote socio-economic development (Mishkin, 2012). There are three reasons to promote financial markets. First is to overcome the mismatch of capital supply and demand. Secondly, the demand for buying, selling and transferring stocks is the growing. Third, with the diversification of commodity economy, it is beneficial to flexible capital flow. The financial markets' primary function is to connect individuals so money can flow where it's needed.

Stock, over-the-counter trading, bond, currencies, derivatives and cryptocurrency markets are all financial markets. The decentralised digital assets, which are cryptocurrencies, are also included in financial markets. Recently, blockchain-based cryptocurrency has gained prominence where there are thousands of crypto-tokens that are traded through online crypto-exchanges throughout the world. Cryptocurrencies only exist in encrypted form. They are stored and traded using software or smart deposit devices. Besides, there is no need for a central organization to manage and maintain the value of cryptocurrency (Ashford & Schmidt, 2022). All transactions of cryptocurrency are conducted online, or through special networks and applications. Today, the core technical element of cryptocurrency is the blockchain system, which makes cryptocurrency unchangeable, decentralized and transparent. Therefore, cryptocurrencies are not issued by the government. In the virtual currency world, there are new blockchain products like decentralised finance, NFT, and the Metaverse that will have the potential to develop future cryptocurrency markets.

The advantages of cryptocurrency is to reduce the transaction costs by eliminating the authoritative organization or third parties. It is convenient to use, especially when paying with cryptocurrency, and it is easy to store cryptocurrency. For example, cryptocurrency is stored in an application, hardware wallet or online storage. Therefore, cryptocurrency will make it easier for both parties to transfer funds (Nakamoto, 2008). Funds are transferred directly from one person to another, and there is no transaction costs. If transaction costs are required, it is only a minimum amount. However, the transaction processing time is less than that of bank transactions, and all transaction information will be confidential and anonymous. In addition, cryptocurrency reduces inflation, because the creator of Bitcoin, Satoshi Nakamoto, set the upper limit at 21 million, so there is no large-scale cloning. When governments issues too much legal tender, it can lead to inflation or devaluation. In addition, cryptocurrency is pre-programmed with algorithms that limit supply and can reduce inflation. Unlike fiat money, cryptocurrencies cannot be counterfeited as cryptocurrencies use blockchain technology with the combination of other consensus mechanisms integrated in algorithms to establish their system. The transparency of cryptocurrency is also an advantage. Blockchain records all encrypted currency transactions and information, and its encryption system and decentralised network can prevent external interference.

Previous studies have analyzed the tail behavior of cryptocurrency by using the EVT method. For example, in the research of Osterrieder and Lorenz (2017), an extreme value analysis of bitcoin returns is provided. They compared the tail risk characteristics of bitcoin with the traditional exchange rates between the US dollar and the G-10 currencies. According to their research, the volatility of Bitcoin is higher than that of the traditional G-10 currencies, and the return distribution of Bitcoin has stronger non-normality and heavier tails. Gkillas and Katsiampa (2018) used peaks over threshold (POT) approach to study the tail behaviour of the daily returns of Bitcoin, Ethereum, Ripple, Bitcoin Cash and Litecoin. They found that bitcoin cash is the most risky cryptocurrency among the five cryptocurrencies, which meant it had the greatest potential loss and gain. However, this study showed that Bitcoin and Litecoin are the lowest risk cryptocurrencies.

Ali et al. (2021) employed the GEV distribution to model the monthly maximum negative log returns of Malaysian gold prices. Throughout the study, the GEV model's parameters were estimated using maximum likelihood estimation (MLE) and L-moments (LMOM) techniques. The researchers also showed the quantile-quantile plot to indicate the accuracy of fitting the GEV model with the data by using MLE. Apart from that, VaR was estimated as the upper quartile of 10%, 5%, and 1% to determine the potential losses of investing in the gold market. Using EVT, the frequency and probability of extreme situations in the financial world are studied, because it focuses on extreme values and can produce a more accurate risk model prediction. A study by Hussain et al. (2021) modelled daily extreme returns in the Bitcoin market by employing GP. The Bitcoin's return levels were estimated by using the daily log losses of Bitcoin and they found that Bitcoin's returns data have heavy tail and finite tail distribution characteristics. Islam and Das (2021) explored and developed EVT's ability to predict bitcoin returns, because EVT was proposed to deal with unusual but extreme events, such as heavy losses or large-scale damage. This study used various statistical tests to demonstrate the extreme nature of Bitcoin's return. However, the main focus of this study was to model the return of Bitcoin by using the POT method and the block maxima (BM) method, and to evaluate the uncertainty by predicting the return level of Bitcoin in the next five, ten, twenty, fifty and one hundred years (with a confidence interval of 95%). The result of this study had shown that the BM approach provided a better fit with Bitcoin's returns data compared to the POT approach.

Although EVT is widely used to investigate and explain the extreme events in the financial field, there is still a lack of research on using EVT to analyse bitcoin, especially in predicting extreme returns of cryptocurrency, because most research focuses on the overall distribution rather than the tail distribution. Apart from that, Hussain et al. (2021) predicted the extreme returns of Bitcoin using GP and found that it was difficult to determine which cryptocurrency provided the highest return and the accuracy of the model in predicting extreme returns, because they only used one cryptocurrency and one model in their research. Meanwhile, Islam and Das (2021) predicted the extreme positive returns of Bitcoin only and suggested using the negative returns to estimate the negative return levels for future work. All the limitations and suggestions for future work provide the impetus for this research to apply the BM method using GEV model and the POT method using GP model to predict the negative extreme returns of Bitcoin and Ethereum.

Several studies have compared the accuracy of different types of models in estimating risk value, and conducted backtesting procedures to determine the accuracy of models in estimating risk value. For instance, Johansson and Nilsson (2011) compared the performance of seven different techniques in estimating the VaR for a portfolio consisting five Swedish index-bonds with different maturities. In their research, three different windows and seven different methods which are basic historical simulation (HS), age-weighted historical simulation (AWHS), volatility-weighted historical simulation (VWHS), normal distribution, Student's t-distribution, asymmetric slope, and symmetric absolute value, were applied to calculate one-day VaR estimates for this portfolio. Kupiec and Christoffersen test for unconditional and conditional coverage was also conducted to evaluate the accuracy of the approaches. In addition, Cao & Johansson (2022) has estimated VaR and Expected Shortfall (ES) of Bitcoin (BTC), Ethereum (ETH), Binance coin (BNB), Ripple coin (XRP), and Cardano (ADA) by using three parametric (normal, Students's t and GP) and three non-parametric estimation methods (HS, AWHS and VWHS).

In this study, we will estimate VaR by using BM and POT methods. The parameters of GEV and GP will be obtained by using L-Moment (LMOM) because it handles the extreme value better. Then, nonparametric and parametric methods will be used to estimate VaR, and Christoffersen backtesting program will be used to evaluate it.

MATERIALS AND METHODS

Data Used

The data used in this study are the daily closing prices of Bitcoin and Ethereum extracted from Yahoo! Finance. The daily data used are in form of US Dollar (USD) which are from 1st January 2020 to 31st December 2022. In addition, the data is extracted according to the BM approach of GEV distribution and POT approach for GP distribution. The logarithmic return for Bitcoin and Ethereum will be calculated. This is because the log-returns can be managed easily and it provides better statistical behaviour. Therefore, the logarithmic return series is widely used in financial research, rather than using the actual prices. The transformation of prices into returns will be done through the following equation:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

where R_t is the return of Bitcoin or Ethereum in period t
P_t is the closing price in period t
P_{t-1} is the price in period $t-1$.

Extreme Value Theory

According to the central limit theorem, when the sample size provided is sufficiently large enough, the sum and the mean of an arbitrary finite distribution are normally distributed. However, in some practical studies, instead of the data's average, actually we are more interested with the maximum or minimum values of the limiting distribution.

Assume that $X_1, X_2, \dots, X_n$ is a sequence of iid random variables having a common distribution function F. One of the most interesting statistics in research is the sample maximum

$$M_n = \max\{X_1, X_2, \dots, X_n\}.$$

This theory studied the behaviour of M_n as the sample size n increases to infinity.

$$\begin{aligned} Pr\{M_n \geq x\} &= Pr\{X_1 \leq x, X_2 \leq x, \dots, X_n \leq x\} \\ &= Pr\{X_1 \leq x\}\, Pr\{X_2 \leq x\} \dots \dots Pr\{X_n \leq x\} \\ &= F_n(x) \end{aligned}$$

Block Maxima Approach

Block maxima (BM) approach is the most popular extreme value approach. According to the BM approach, data are divided into blocks of equal length. For example, in years, months or days. Then, the highest observation of each block is analysed to fit into a model. In this study, the data was fitted into the model using BM approach by dividing the time series data into monthly blocks.

Generalized Extreme Value Distribution

Jenkinson (1955) proposed a formula for Generalized Extreme Value (GEV) Distribution which is a single parametric family that encompasses the three types of limiting distribution of Gnedenko (1943). It comprises the Gumbel distribution, known as EVI distribution, the Fréchet distribution, also known as EVII distribution, and the Weibull distribution, also known as EVIII distribution.

The cumulative distribution function (CDF) of GEV distribution is defined as:

$$F(x,\mu,\sigma,\xi)=\begin{cases}\exp\left[-\left(1+\dfrac{\xi(x-\mu)}{\sigma}\right)^{-\frac{1}{\xi}}\right] & \text{if } \xi\neq 0,\\ \exp\left[-e^{-\frac{x-\mu}{\sigma}}\right] & \text{if } \xi=0\end{cases} \tag{1}$$

where the scale parameter is $\sigma > 0$, location parameter is $-\infty < \mu < \infty$ and ξ as the shape parameter that represents the behaviour of the tail.

The sub-models can be defined by

• Type I: $\xi = 0$, the Gumbel family with CDF

$$F(x) = \exp\left(-e^{-\frac{x-\mu}{\sigma}}\right),\ x \in \mathbb{R}$$

• Type II: $\xi > 0$, the Fréchet family with CDF

$$F(x)=\begin{cases}\exp\left[-\left(\dfrac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right], & x>\mu, \dfrac{1}{\xi}>0,\\ 0, & x\leq 0\end{cases}$$

• Type III: $\xi < 0$, the Weibull family with CDF

$$F(x)=\begin{cases}\exp\left[-\left(-\dfrac{x-\mu}{\sigma}\right)^{\frac{1}{\xi}}\right], & x<\mu, \dfrac{1}{\xi}>0,\\ 1, & x\leq 0\end{cases}$$

for the parameters $\sigma > 0, -\infty < \mu < \infty$.

Peaks Over Threshold Approach

In peaks-over-threshold (POT) approach, a threshold for the data will be chosen. However, only data that lies above the threshold value will be selected for fitting into the model. Therefore, it is necessary to select a threshold that is neither too high nor too low in order to conform to the GP function. If the threshold is not high enough, there is a chance of getting biased estimations. Meanwhile, if the threshold is too high, there will only be a few data that are available for analysis, which will lead to a higher variance of the estimates.

Generalised Pareto Distribution

The Generalised Pareto (GP) distribution was introduced by Pickands III (1975), and Balkema and De Haan (1974). GP is a limiting distribution of the standardized excesses over a threshold, as the threshold approaches the endpoint of the variable. The distribution function for GP can be written as

$$F(x,\mu,\sigma,\xi)=\begin{cases}1-\left(1+\dfrac{\xi(x-\mu)}{\sigma}\right)^{-\frac{1}{\xi}} & \text{if } \xi\neq 0,\\ 1-e^{-\frac{x-\mu}{\sigma}} & \text{if } \xi=0.\end{cases}$$

for $x\geq 0$, when $\xi\geq 0$, or $x\geq 0$, and $x\leq -\frac{\sigma}{\xi}$ when $\xi<0$, where $\sigma>0$ is the scale parameter and $\xi\in\mathbb{R}$ is the shape parameter.

The shape parameter of the GP play a dominant role in determining the qualitative behaviour of the tail. So, the following values of the parameter ξ are of interest:

- When $\xi\to 0$, the GP converges to the exponential distribution with mean σ.
- When $\xi=-1$, the GP becomes the uniform distribution $U(0,\sigma)$.
- When $\xi=\frac{1}{2}$, the GP becomes the triangular distribution.
- The Pareto distribution is obtained when $\xi>0$.

Apart from that, GP has associated a Pareto type-II model if the parameter ξ < 0 is related to long tail behaviour. However, it will follow short tail distribution if ξ > 0.

Parameter Estimation

In this study we will use L-moment and maximum likelihood estimation methods to investigate the effectiveness of L-moment and maximum likelihood estimation in estimating parameters.

L-moment estimation

L-moment is based on the probability weighted moments describe the shape of probability distributions. The method can be defined as the linear combination of probability weighted moments (Hosking 1990). The L-moment of order r can be defined as:

$$\lambda_r=\frac{1}{r}\sum_{i=0}^{r-1}(-1)^i\binom{r-1}{i}E(Y_{r-1:r})\qquad \text{for } r=1,2,\ldots$$

The GEV distribution parameters according to the L-moment method can be described as follows:

$$\hat{\mu}=\hat{\lambda}_1+\frac{\hat{\sigma}\left[\Gamma\left(1+\hat{\xi}\right)-1\right]}{\hat{\xi}}$$

$$\sigma=\frac{\hat{\lambda}_2\hat{\xi}}{\Gamma\left(1+\hat{\xi}\right)\left(1-2^{-\hat{\xi}}\right)}$$

$$\hat{\xi}=7.8590\hat{c}+2.9554\hat{c}^2$$

where μ, σ and ξ are the location, scale and shape parameters respectively; λ_1, λ_2 and λ_3 are the L-moment and

$$\hat{c}=\frac{2\hat{\lambda}_2}{\hat{\lambda}_3+3\hat{\lambda}_2}-\frac{\ln(2)}{\ln(3)}$$

On the contrary, the estimates of GP parameters are

$$\hat{\sigma} = \hat{\lambda}_1\left(1 + \hat{\xi}\right)$$

$$\hat{\xi} = \frac{\hat{\lambda}_1}{\hat{\lambda}_2} - 2$$

where σ and ξ are the scale and shape parameters respectively and the first and the second samples of L-moment are

$$\hat{\lambda}_1 = \frac{\hat{\sigma}}{1 + \hat{\xi}}$$

$$\hat{\lambda}_2 = \frac{\hat{\sigma}}{(1 + \hat{\xi})(2 + \hat{\xi})}$$

Maximum Likelihood Estimation

From the GEV distribution function, the probability distribution function is

$$f_X(x) = \frac{1}{\sigma}\left(1 + \frac{\xi(x-\mu)}{\sigma}\right)^{-\left(1+\frac{1}{\xi}\right)} \exp\left[-\left(1 + \frac{\xi(x-\mu)}{\sigma}\right)^{-\frac{1}{\xi}}\right]$$

For a set of n observations, the log-likelihood function of an observation x_j is

$$l\left(x_j; \hat{\mu}, \hat{\sigma}, \hat{\xi}\right) = -n \ln \hat{\sigma} - \left(1 + \frac{1}{\hat{\xi}}\right)\sum_{j=1}^{n} \ln\left[1 + \frac{\hat{\xi}\left(x_j - \hat{\mu}\right)}{\hat{\sigma}}\right] - \sum_{j=1}^{n} \ln\left[1 + \frac{\hat{\xi}\left(x_j - \hat{\mu}\right)}{\hat{\sigma}}\right]^{-\frac{1}{\hat{\xi}}}$$

and the maximum likelihood estimates of the location, μ, the scale, σ and the shape, ξ, parameters are obtained by maximising the log likelihood.

In contrast, from the GP distribution function, the probability distribution function of GP for $\xi \neq 0$ is

$$f_X(x) = \frac{1}{\sigma}\left(1 + \frac{\xi(x-\mu)}{\sigma}\right)^{-\left(1+\frac{1}{\xi}\right)}$$

For a set of k observations, the log-likelihood function of an observation x_i is

$$l\left(x_i; \hat{\sigma}, \hat{\xi}\right) = -k \ln \hat{\sigma} - \left(1 + \frac{1}{\hat{\xi}}\right)\sum_{i=1}^{k} \ln\left[1 + \frac{\hat{\xi}(x_i - \hat{\mu})}{\hat{\sigma}}\right]$$

and the maximum likelihood estimates of the scale, σ and the shape, ξ, parameters are obtained by minimising the negative log-likelihood function.

Simulation Studies

Extensive simulation studies are conducted in this section to investigate the performance of estimation of σ and ξ using L-moment (LMOM) and maximum likelihood estimation (MLE). The evaluation of the performances will be based on the empirical bias, mean square error (MSE) and root mean square error (RMSE) estimated from 10000 simulations. The simulations are conducted as follows:

1. The negative daily log-returns of Bitcoin and Ethereum are fitted into the GP model.

2. Parameter estimation using LMOM is conducted to obtain the parameters values for Bitcoin and Ethereum.
3. Independent and identically distributed (i.i.d) observations are generated following $\mathrm{GP}(\sigma, \xi)$
4. Two different pairs of parameters are used where the first pair of parameters are obtained from the parameter estimation of Bitcoin while the second pair of parameters are obtained from the parameter estimation of Ethereum.
5. LMOM and MLE are used to estimate σ and ξ.
6. The steps above are repeated 10000 times and n is varied between 500, 5000 and 50000 to compute the bias, MSE and RMSE of σ and ξ.

Goodness-of-Fit

Goodness-of-fit test statistics are used for checking the validity of a specified or assumed probability distribution model. The Anderson-Darling test is applied in this section.

Anderson-Darling (AD) Test

Anderson-Darling (AD) test is one of the most powerful empirical distribution function (EDF) test. It was first introduced by Anderson and Darling to place more weight at the tails of the distribution (Farrell and Stewart, 2006). In cases with relatively large extremes, it may be expected the AD test to be more suitable to select the best-fitted model to data maxima. The AD test statistic, the quadratic class of the EDF test statistic, is expressed as A^2 as follows:

$$A^2 = -n - \sum_{i=1}^{n} \left[\frac{2i-1}{n} \left(\ln F_X(x_i) + \ln\left(1 - F_X(x_{n+1-i})\right) \right) \right]$$

where $F_X(x_i)$ is the cdf of the proposed distribution at x_i, for $i = 1, 2, \ldots, n$.

The observed data must be arranged in increasing order, as $x_1 < x_2 < \cdots < x_n$. On the other hand, the AD test gives more weight to the tails. Hnece, it is a more accurate test when the tails of the selected theoretical distribution are the focus of the analysis, as with extreme data.

Value-at-Risk

Value-at-Risk estimation is conducted for both cryptocurrencies using a non-parametric method and a parametric method.

I. Non-Parametric Method (age-weighted historical simulation)

The non-parametric approach will use historical data to calculate VaR. It does not make any assumption on the past data and it mainly depends on the historical simulation method. Boudoukh, Richardson & Whitelaw (1998) suggested weighting the observations according to their age in this approach. They assigned higher weights for most recent observations as follows, where w_1 is the weight for the newest observation:

$$w_1 = \frac{1 - \lambda}{1 - \lambda^n}$$

$$w_i = \lambda^{i-1} w_1$$

Constant λ lies between 0 and 1 and reflects exponential rate of decay. For the special case $\lambda \to 1$, age-weighted historical simulation (AWHS) converges to basic historical simulation. Good summary of improvement of age-weighting against basic historical simulation is given in Dowd (2005) and mentioned that there are few major attractions of AWHS which are providing a better generalization of traditional historical simulation, a suitable choice of λ that can make VaR estimates more responsive to large loss observations and makes them better at handling clustering of large losses. In addition, age-weighting helps to reduce distortions caused by events that are unlikely to recur and reduces ghost effects. Older observations will probably lose their probability weights and their power to influence current VaR falls over time. Last but not least, AWHS can be more effective as it gives the option of letting the sample grow with time.

2. Parametric Method (Normal distribution)

The parametric approach is also known as the analytic or correlation method. When there are large numbers of assets in a portfolio, the easiest way to estimate VaR is by using parametric approach. A basic VaR model can be based on the normal distribution which requires only the mean and standard deviation in order to model the distribution. The normal VaR can be estimated by using the following equation

$$\text{VaR} = \mu - \sigma z$$

where μ is the mean, σ is the standard deviation and z refers to the z-score corresponding to the 95% confidence level. The normal distribution is widely used because of its simplicity which only requires two independent parameters, mean and standard deviation.

Backtesting

VaR models are valuable only if they can predict the future risks accurately. Hence, the models should undergo the process of backtesting to test the efficacy of the VaR models and to determine whether the VaR models are adequate or not. In this study, the backtesting method, three-stage unconditional coverage and independence test by Christoffersen is performed.

Violation process, also known as hit sequence is a popular backtesting procedure. The "hit sequence" of VaR violations is defined as

$$I_{t+1} = \begin{cases} 1, & r_{t+1} < -VaR_{t+1}^{\alpha} \\ 0, & r_{t+1} \geq -VaR_{t+1}^{\alpha} \end{cases}$$

where VaR_{t+1}^{α} is the VaR prediction at time $t+1$ for risk quantile level α. The hit sequence will return 1 if the loss in day $t+1$ exceeds the predicted VaR number, else return 0.

When performing a backtest, a sequence $\{I_{t+1}\}_{t+1}^{T}$ across T days that indicates the past violations occur will be constructed. For violation prediction, the hit sequence, $\{I_{t+1}\}_{t+1}^{T}$ is a sequence of iid Bernoulli random variables with null hypothesis

$$H_0 : I_{t+1} \sim \text{Bernoulli}(\alpha)$$

where the Bernoulli variable takes value of 1 with probability p and value of 0 with probability $(1-p)$. The Bernoulli distribution is shown below:

$$f(I_{t+1}; p) = p^{I_{t+1}}(1-p)^{1-I_{t+1}}$$

Unconditional Coverage Test

The objective of unconditional coverage test is to determine the fraction of observed violations for a particular risk model π that is significantly different from the coverage rate, p. The null hypothesis and alternative hypothesis are

$$H_0: p = \pi$$
$$H_1: p \neq \pi$$

The likelihood function for the null hypothesis is

$$L(p) = (1-p)^{T_0} p^{T_1}$$
$$L(\pi) = (1-\pi)^{T_0} \pi^{T_1}$$

Then, the maximum likelihood estimator of $\hat{\pi} = \frac{T_1}{T}$ is estimated. The maximised likelihood for the sample is then given by

$$L(\hat{\pi}) = \left(\frac{T_1}{T}\right)^{T_1} \left(\frac{T_0}{T}\right)^{T_0}$$

The null hypothesis can be tested by means of the following likelihood ratio test:

$$LR_{UC} = -2\ln\left(\frac{L(p)}{L(\pi)}\right) \sim \chi^2(1)$$

Under the null hypothesis that the VaR model is correct, LR_{UC} is asymptotically chi-square distributed with one degree of freedom. However, this test focuses only on the number of exceptions. The null hypothesis which is the VaR model gives the correct coverage rate is rejected when the p-value is less than the desired significance level. Christoffersen recommends using a p-value of 0.1 on the count of type II errors which is being costly in practice.

Independence Test

The independence test is used to investigate whether the violations are independent of one another. The hit sequence $\{I_t\}_{t=1}^{T}$ is assumed to be independent over time t and be described by a discrete-time Markov chain with transition matrix of π_1.

$$\pi_1 = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix} = \begin{bmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{11} \end{bmatrix}$$

where $\pi_{ij}(i,j \in \{0,1\}) = P(I_{t+1} = j | I_t = i)$. Meanwhile, π_{01} is the probability of violation occurs tomorrow given that no violation occurs today and π_{11} is the probability of violation occurs tomorrow given that no violation occurs today.

The likelihood function for this Markov process with T observations as follows

$$L(\pi_1) = (1-\pi_{01})^{T_{00}} \pi_{01}^{T_{01}} (1-\pi_{11})^{T_{10}} \pi_{11}^{T_{11}}$$

where T_{ij} is the number of days with an i followed by a j occurred in the hit sequence, with $i, j \epsilon \{0, 1\}$.

Then, the maximum likelihood estimates for π_{01} and π_{11} are then given by

$$\hat{\pi}_{01} = \frac{T_{01}}{T_{00} + T_{01}}, \quad \hat{\pi}_{11} = \frac{T_{11}}{T_{10} + T_{11}}$$

Hence, the estimated transition matrix will be

$$\pi_1 = \begin{bmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{11} \end{bmatrix} = \begin{bmatrix} \frac{T_{00}}{T_{00} + T_{01}} & \frac{T_{01}}{T_{00} + T_{01}} \\ \frac{T_{10}}{T_{10} + T_{11}} & \frac{T_{11}}{T_{10} + T_{11}} \end{bmatrix}$$

If the hit sequence is independent over time, $\hat{\pi}_{01}$ will be assumed to be equal to $\hat{\pi}_{11}$ and $\hat{\pi}$ which follow the null hypothesis in independent test:

$$H_0 : \pi_{01} = \pi_{11} = \pi$$

and the transition probability matrix will be in the form:

$$\hat{\pi} = \begin{bmatrix} 1-\hat{\pi} & \hat{\pi} \\ 1-\hat{\pi} & \hat{\pi} \end{bmatrix}$$

Then, to test the independence hypothesis, the likelihood ratio test statistics defined as follows:

$$LR_{IND} = -2\ln\left(\frac{L(\hat{\pi})}{L(\hat{\pi}_1)}\right) \sim\chi^2(1)$$

Conditional Coverage Test

The conditional coverage test is the combination of unconditional coverage test and independence test to be tested jointly whether the average number of violations is correct and the hit sequence is independent. The likelihood ratio test is shown as:

$$LR_{CC} = LR_{UC} + LR_{IND} = -2\ln\left(\frac{L(p)}{L(\hat{\pi})}\right) - 2\ln\left(\frac{L(\hat{\pi})}{L(\hat{\pi}_1)}\right) = -2\ln\left(\frac{L(p)}{L(\hat{\pi}_1)}\right) \sim\chi^2(2)$$

RESULTS AND DISCUSSION

Data and Descriptive Analysis

In this research, the daily adjusted closing prices of Bitcoin (BTC) and Ethereum (ETH) are used. The data is sourced from Yahoo! Finance, with a total of 1096 observations from 1st January 2020 to 31st December 2022. The daily adjusted closing prices are converted into daily log-returns and the preliminary descriptive analysis of Bitcoin and Ethereum are shown in Figure 1. Descriptive statistics provide the measures and the summaries of samples, hence, it is an important part in describing the fundamental features of the sample data used in research. The quantitative data is described using measures of central tendency and dispersion. The descriptive statistics of daily return for Bitcoin and Ethereum are shown in Table 1.

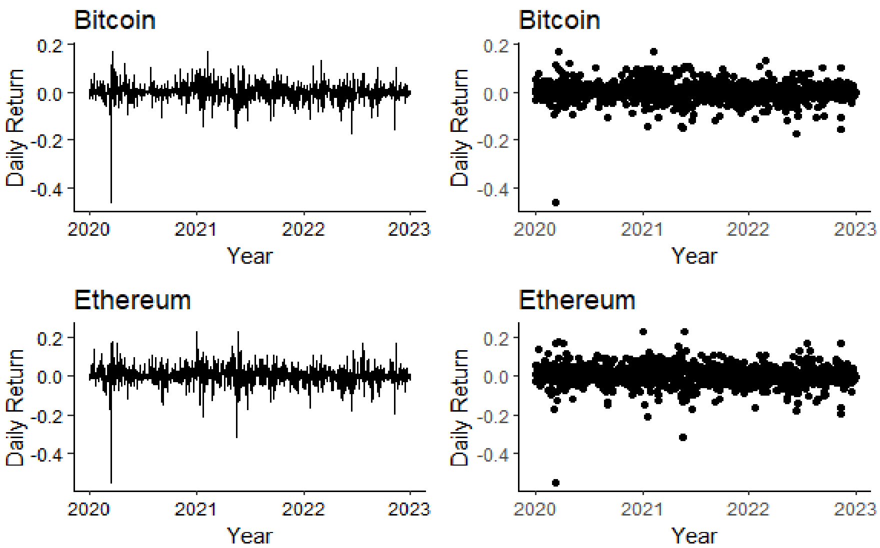

Figure 1: Time series plots and scatter plots of Bitcoin and Ethereum daily return in the period between 1st January 2020 to 31st December 2022

Table 1: Descriptive summary statistics of Bitcoin and Ethereum daily return

Cryptocurrencies	Mean	Median	Standard Deviation	Skewness	Kurtosis	Minimum	Maximum
Bitcoin	0.0008	0.0008	0.0388	−1.6365	20.4395	−0.4647	0.1718
Ethereum	0.0020	0.0029	0.0517	−1.3579	14.4169	−0.5507	0.2307

From Table 1, some interesting conclusions can be drawn on these two cryptocurrencies. The mean daily log-return for both cryptocurrencies is positive and the standard deviations are relatively small. This shows that both cryptocurrencies have brought slightly increasing positive return. The mean and standard deviation of bitcoin are 0.0008 and 0.0388 respectively. The lowest daily return of Bitcoin is 46.47% and the highest daily return is 17.18%. Meanwhile, the mean for Ethereum is a positive value of 0.0020 and its standard deviation is 0.0517. Ethereum's lowest daily return is 55.07% and the highest daily return is 23.07%. Hence, Ethereum's daily gain and daily loss are greater than Bitcoin during the relevant period. Apart from that, skewness and kurtosis are important in this research as they are the measures of symmetry and "tailedness" of a distribution. The skewness and kurtosis of Bitcoin are -1.6365 and 20.4395 whereas the skewness and kurtosis of Ethereum are -1.3579 and 14.4169. As a result, both cryptocurrencies are negatively skewed and have the potential to display a heavy-tail behaviour as both skewness are negative and kurtosis are greater than 3.

In Figure 2, it shows that the Bitcoin daily return is negatively skewed due to the presence of a long tail in the negative direction on the horizontal axis. From the histogram, the Bitcoin daily return is asymmetric and has a fatter tail in the negative direction. Besides that, there are some extreme values which are called outliers in the box-and-whiskers plot. For some studies, researchers might remove the outliers. However, the extreme values will not be excluded from this research. In Figure 3, it shows that the Ethereum daily return is negatively skewed and outliers are present in the box-and-whiskers plot. From the histogram, it can be seen that the Ethereum daily return is asymmetric and it is left-skewed which is known as a negative-skewed distribution. The fatter tail in the negative direction indicates that Ethereum gives a large number of negative returns.

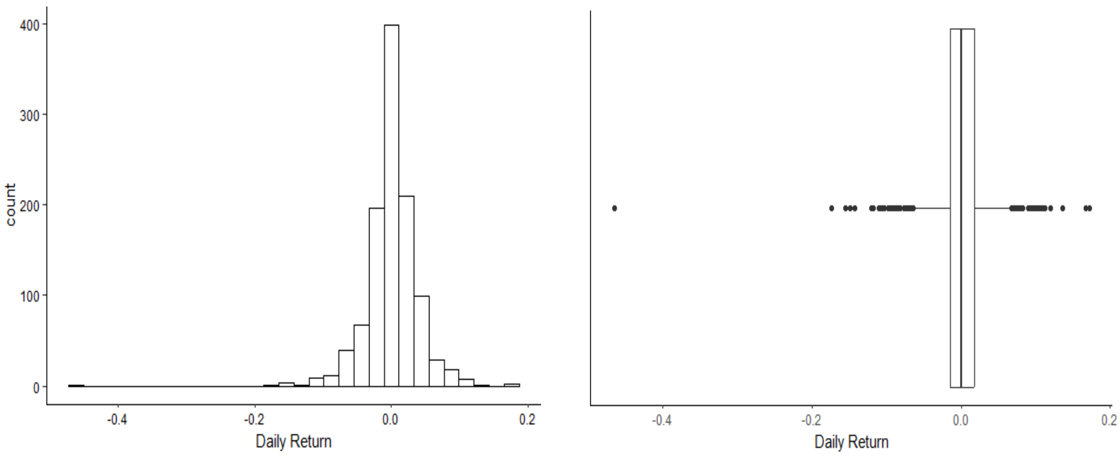

Figure 2: Histogram and box-and whiskers plot of Bitcoin daily return

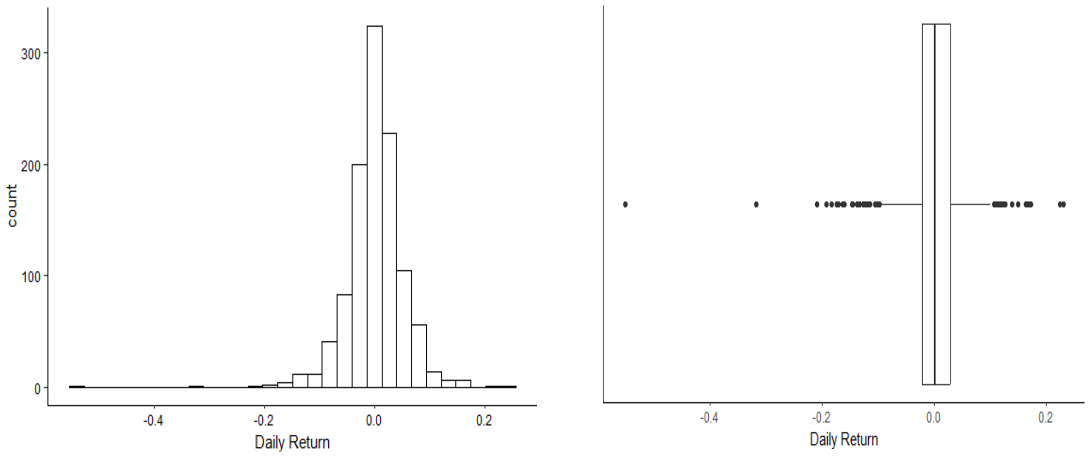

Figure 3: Histogram and box-and whiskers plot of Ethereum daily return

Normality Test

Normality test is an essential step in analysing continuous data. It helps researchers to decide the measures of central tendency and statistical methods for data analysis. In this study, the Shapiro-Wilk normality test is used to check the normality assumption of the data and validate the claim that the cryptocurrencies' daily log- return data are not normally distributed. The test hypothesis of this test is as follow:

H_0: The log-return series are normally distributed

H_a: The log-return series are not normally distributed

The result of Shapiro-Wilk Test for cryptocurrencies daily return is presented in Table 2.

Table 2: Shapiro-Wilk Test for Bitcoin and Ethereum

Cryptocurrencies	Test Statistics (W)	p-value
Bitcoin	0.88751	$< 2.2 \times 10^{-16}$
Ethereum	0.90823	$< 2.2 \times 10^{-16}$

The null hypothesis is rejected for the Shapiro-Wilk test because the p-value of Bitcoin and Ethereum tabulated are very small, which is lesser than $\alpha = 0.05$. The rejection of the null hypothesis concluded that the log-return data of Bitcoin and Ethereum are not normally distributed. Besides that, a graphical approach which is Q-Q plot is implemented to substantiate the result of the Shapiro-Wilk Test.

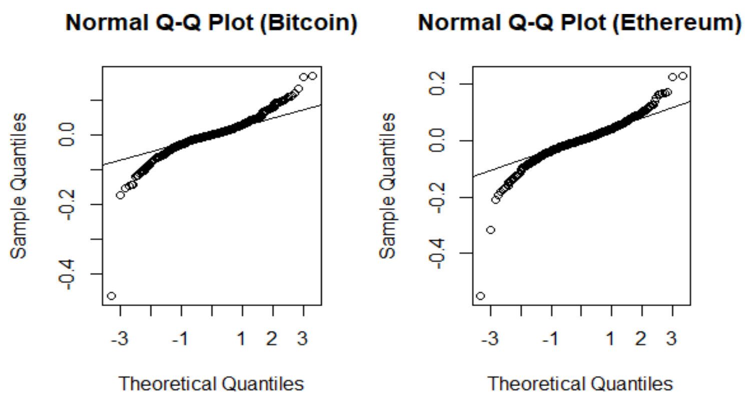

Figure 4: Normal Q-Q Plot for Bitcoin and Ethereum daily return

Stationarity Test

The concept of stationarity is important in time series forecasting as a stationary time series data will provide a better prediction of future returns. Therefore, it is essential to perform stationarity tests in studies where the underlying variables are time-based. In this research, Augmented Dickey-Fuller (ADF) test is conducted to test the stationarity of the cryptocurrencies' daily log-return series. The null hypothesis and alternative hypothesis are shown as follow:

H_0 : The series is non-stationary

H_a : The series is stationary

Table 3: Augmented Dickey-Fuller test result

Cryptocurrencies	Test Statistics (W)	p-value
Bitcoin	-9.6976	0.01
Ethereum	-9.7773	0.01

The result of the ADF test is reported in Table 3. Since the p-value for both log-return series is equal to 0.01, hence the null hypothesis is rejected at $\alpha = 0.05$. A conclusion can be made from this Augmented Dickey- Fuller test which is the log-return series of Bitcoin and Ethereum are stationary.

Block Maxima Approach and Peaks Over Threshold Approach

From the descriptive statistics obtained, it is clear that the distribution is negatively skewed and the kurtosis is much higher than the kurtosis of normal distribution. It indicates that the extreme outcomes are more frequent and the number of daily log losses is greater than the daily log returns. Hence, this study will focus on the negative returns which are related to the downside risk.

In this study, the minimum daily log-return which is also known as negative daily log-return is implemented for predicting the maximum extreme returns through the GEV and GP model. However, for easier understanding and tabulating the data, the negative returns are transformed into positive ones. For block maxima approach, the negative log-return series is divided into monthly blocks. Then, the monthly maxima of negative log-return of the cryptocurrency's prices are modelled using the GEV distribution.

For the peaks over threshold approach, the optimal threshold is chosen by using the package *fExtremes* in R software because choosing the threshold value from the mean residual life plot is very subjective. The threshold value simulated by R for the negative log- return of Bitcoin is 0.0591 while the threshold value simulated for the negative log- return of Ethereum is 0.0753. After that, the data that exceeds the threshold value will only be fitted into the GP model.

Parameter Estimation

A statistical approach known as parameter estimation is used to estimate the parameter values of the sample data in a statistical model. The objective is to determine the parameter values that best fit the sample data and produce the most accurate forecasts. In this section, the parameters of the Generalized Extreme Value (GEV) Distribution and Generalized Pareto Distribution (GP) are estimated through L-moment estimation. The estimation of GEV and GP models fitting are implemented in R.

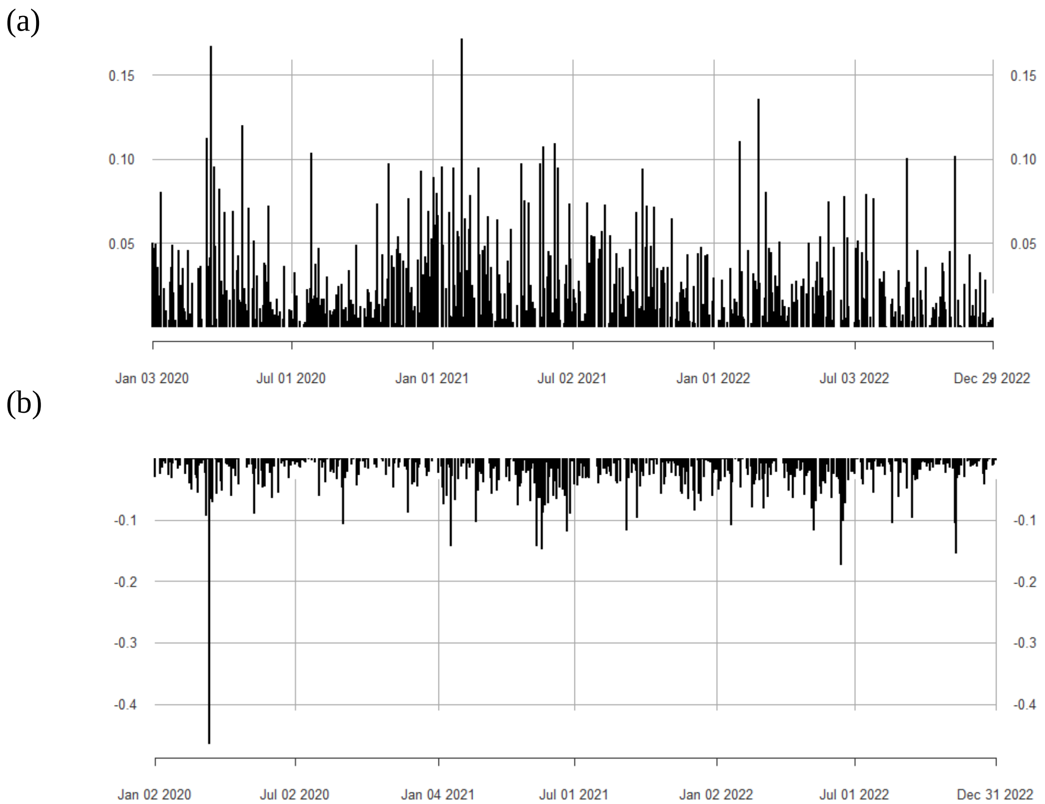

Figure 5: The maximum and minimum daily log-return of Bitcoin: (a) positive returns and (b) negative returns

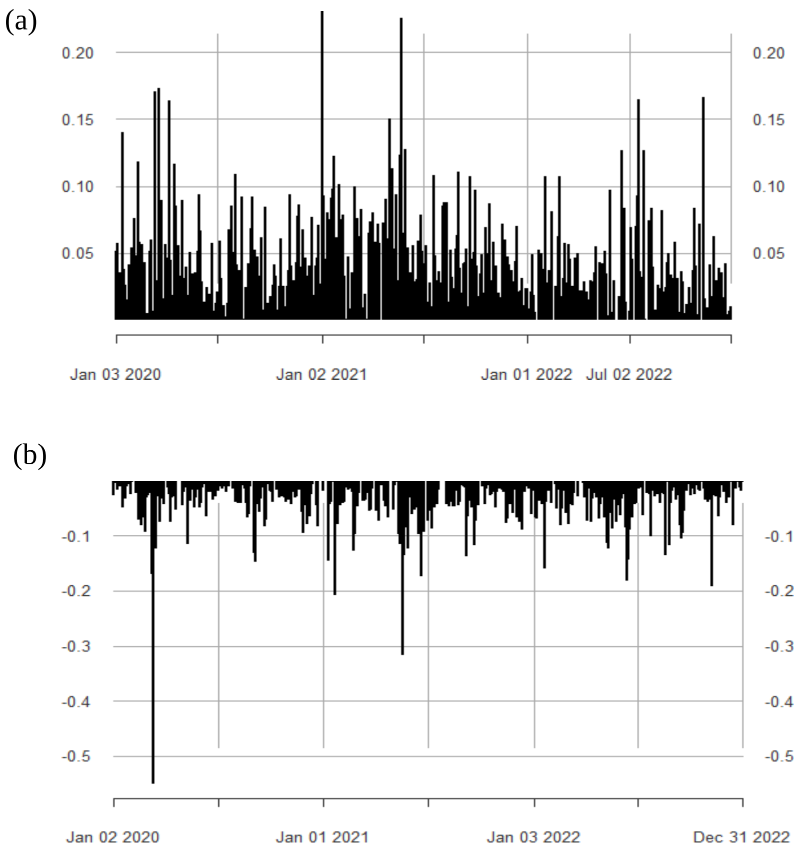

Figure 6: The maximum and minimum daily log-return of Ethereum: (a) positive returns and (b) negative returns

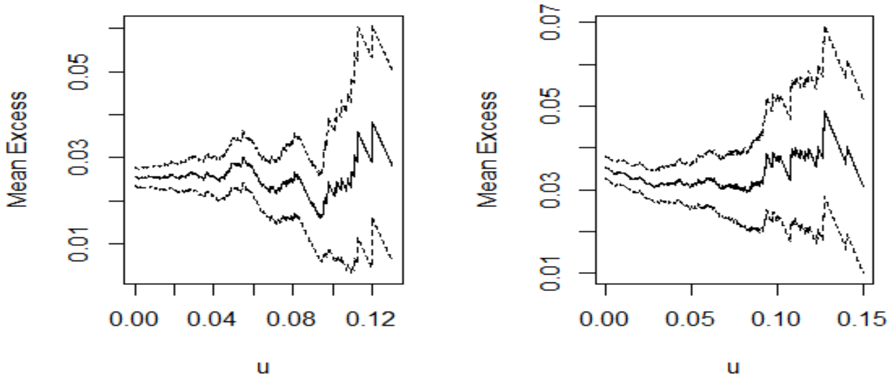

Figure 7: Mean residual life plot for Bitcoin (left) and Ethereum (right)

Generalized Extreme Value (GEV) Distribution

The parameters of Generalized Extreme Value distribution which are the location parameter (μ), scale parameter (σ) and shape parameter (ξ) are estimated via L- moment estimation. The results for both Bitcoin and Ethereum are shown in Table 4.

Table 4: The estimated parameters of GEV distribution for Bitcoin and Ethereum

Cryptocurrencies	Parameters		
	μ	σ	ξ
Bitcoin	0.060546	0.033243	0.267894
Ethereum	0.074160	0.038924	0.345114

The shape parameter, ξ obtained from both negative daily log-return are positive and are significantly different from zero at a 5% asymptotic level. It implies that the negative daily log-return of both cryptocurrencies may have a heavier left tail. Overall, the finding indicates that the distributions of negative daily log-return of Bitcoin and Ethereum belong to the Fréchet family.

Generalized Pareto (GP) Distribution

The parameters of Generalized Pareto distribution are also estimated through L-moment estimation. The scale parameter (σ) and shape parameter (ξ) for both cryptocurrencies are estimated and presented in Table 5. The shape parameter, ξ tabulated from both negative daily log-return are positive. When the shape parameter obtained is positive, the tail of the distribution will become progressively shorter as a polynomial and it is known as an infinite tail. The presence of an infinite tail can result in significant risk for financial portfolios and lead to large losses. Apart from that, the scale value of Ethereum is higher than the scale value of Bitcoin, it can be concluded that the uncertainty of returns of Ethereum is higher because the distribution's spread is obtained from the scale values.

Table 5: The estimated parameters of GP distribution for Bitcoin and Ethereum

Cryptocurrencies	Parameters	
	σ	ξ
Bitcoin	0.022356	0.355622
Ethereum	0.034072	0.302534

Simulation Studies

In the simulation studies, two parameter estimation approaches which are L-moment estimation and maximum likelihood estimation (MLE) are conducted to investigate the estimation of σ and ξ. The extreme events are simulated from Generalized Pareto Distribution (GP) with three different sample sizes, $n = 500$, 5000 and 50000. All evaluations are based on the bias, mean square error (MSE) and root mean square error (RMSE) estimated from 10000 simulations.

The first simulation follows the GP distribution, $X \sim GP(0.022, 0.356)$ where the scale and shape parameters are obtained from the parameter estimation of the Generalized Pareto Distribution of Bitcoin. Meanwhile, the second simulation follows the GP distribution, $Y \sim GP(0.034, 0.303)$ where the scale and shape parameters are obtained from the parameter estimation of the Generalized Pareto Distribution of Ethereum. Table 6 presents the full simulation results for 3 different sample sizes.

Table 6: Bias, MSE and RMSE of GP parameter estimation

GP(σ,ξ)	n	Methods	Bias		MSE		RMSE	
			σ	ξ	σ	ξ	σ	ξ
GP (0.022,0.356)	500	L-moment	−0.0224	−0.3782	0.0005	0.1430	0.0224	0.3782
		MLE	0.0212	−0.0304	0.00046	0.0281	0.0214	0.1677
	5000	L-moment	−0.0224	−0.3780	0.0005	0.1429	0.0224	0.3780
		MLE	0.0212	−0.0030	0.00046	0.0024	0.0213	0.0491
	50000	L-moment	−0.0224	−0.3780	0.0050	0.1429	0.0224	0.3780
		MLE	0.0210	−0.00045	0.00044	0.00024	**0.0211**	**0.0156**
GP (0.034,0.303)	500	L-moment	−0.0341	−0.3366	0.0012	0.1133	0.0341	0.3366
		MLE	0.0190	−0.0187	0.00042	0.0151	0.0206	0.1230
	5000	L-moment	−0.0341	−0.3369	0.0012	0.1135	0.0341	0.3369
		MLE	0.0180	−0.0019	0.00033	0.0014	0.0181	0.0374
	50000	L-moment	−0.0341	−0.3366	0.0012	0.1133	0.0341	0.3366
		MLE	0.0179	−0.00030	0.00032	0.00015	**0.0179**	**0.0123**

Based on Table 6, the smallest values of bias, mean square error (MSE) and root mean square error (RMSE) are identified. Specifically, the scale parameter, σ and shape parameter, ξ yielded the best performance in MLE for both $\text{GP}(0.022, 0.356)$ and $\text{GP}(0.034, 0.303)$. In addition, the values of bias, MSE and RMSE for MLE will reduce when the sample size increases, while remain the same for LMOM. Therefore, it can be concluded that the MLE with sample size 50000 is superior and it provides better estimates of the parameters and less computation time compared to LMOM.

Goodness-of-Fit

Goodness-of-fit for negative daily log-return of Bitcoin and Ethereum are then examined by using

Anderson-Darling (AD) test to evaluate the performance between GEV and GP model. The high p-value in the Anderson-Darling test indicates a better fitting model. The results tabulated are given in Table 7.

Table 7: Anderson-Darling test for Bitcoin and Ethereum

Model	Bitcoin		Ethereum	
	Anderson-Darling	p-value	Anderson-Darling	p-value
GEV	0.2635	**0.6797**	0.2164	**0.8319**
GP	0.2658	0.6786	0.3852	0.3806

For Bitcoin, the GEV distribution gives the highest p-value which is 0.6797 with AD (0.2635) in the goodness of fit test which means GEV distribution fits the data well. Similarly, it can be seen that GEV distribution fits well to the negative daily log-return of Ethereum because of the high p-value shown in Anderson-Darling test. Consequently, GEV distribution fits the two negative log-return series well especially for Ethereum with the highest p-value which is 0.8319.

Value-at-Risk

In this section, age-weighted historical simulation method and normal distribution are used to calculate the VaR of the daily log-return series for Bitcoin and Ethereum. The sample data is separated into in-sample and out-sample data before the VaR is estimated. The sample period for this study is from 1st January 2020 until 31st December 2022. The in-sample period will start from 1st January 2020 to 31st December 2021 while the out-sample period is from 1st January 2022 to 31st December 2022. 95% VaR are tabulated using the in-sample data which consists of 731 business days.

Table 8: A 95% VaR Estimates

Cryptocurrencies	**Non-Parametric Method** Age-weighted historical simulation (AWHS)	**Parametric Method** Normal distirbution
Bitcoin	-6.22%	-6.49%
Ethereum	-7.03%	-8.46%

The VaR at a 95% confidence interval is the daily return located at the 5% percentile of the daily return series. For the non-parametric method which is the age-weighted historical simulation (AWHS), the risk value realized in Bitcoin is -6.22% but is -7.03% for Ethereum. This indicates that the loss for Bitcoin will not exceed 6.22% for 95 days out of 100 days and the loss for Ethereum will not exceed 7.03%. Hence, Bitcoin has a smaller VaR, so it will experience a smaller loss compared to Ethereum and it can be considered as a lower risk cryptocurrency. For the parametric method, which is normal distribution, Ethereum remains as the riskiest cryptocurrency which will give the largest loss of 8.46%. However, the maximum loss that will be experienced by Bitcoin is only 6.49%. This can be easily understood by the following example: if investors make a RM1000 investment on Ethereum, the maximum loss that they will need to face is RM84.60, but if they invest RM1000 on Bitcoin, their loss will not exceed RM64.90. Comparing the two cryptocurrencies, the estimated 95% VaR by using AWHS is lower than normal distribution. Besides, in nonparametric and parametric methods, the VaR value of Bitcoin is lower than that of Ethereum. In fact, because the risk level of Bitcoin is relatively low, it is more worthy of recommendation by investors.

Backtesting

The backtest procedure which consists of unconditional coverage test, independence test and conditional test is conducted in this section. Before the backtest procedure is conducted, VaR for the year 2022 is tested by using the daily cryptocurrencies returns from the year 2020 and 2021 to ensure its accuracy. After that, the estimated VaR is compared with the actual return. The obtained results are discussed in the section below.

VaR-breaks Observations

The number of 95%-VaR produced by age-weighted historical simulation and normal distribution are compared to the expected number of violations at a 95% confidence level. The results are recorded in the table below.

Table 9: Expected and actual number of 95%-VaR obtained

	Bitcoin	Ethereum
Trading days	731	731
Expected $X_{VaR}(0.05)$	16	16
Age-weighted Historical Simulation Actual $X_{VaR}(0.05)$	3	4
Normal distribution Actual $X_{VaR}(0.05)$	15	16

It can be clearly seen from Table 9 that the actual VaR exceedance of normal distribution is equivalent or nearly equal to the expected VaR exceedance whereas the age-weighted historical simulation method overestimated the VaR, as the actual VaR exceedance is lower than the expected VaR violation.

Unconditional Coverage Test

The unconditional coverage test is conducted in this section to determine whether the expected violations, p, are equivalent to the actual violations in the series. The result is shown in Table 10.

Table 10: Unconditional coverage test for 95%-VaR

	Bitcoin	Ethereum
Age-weighted Historical Simulation		
LR_{UC}	17.6824	14.4688
p-value	0.000026	0.0001425
Conclusion	Reject	Reject
Normal distribution		
LR_{UC}	0.1883	0.0313
p-value	0.6643	0.8596
Conclusion	Fail to reject	Fail to reject

As can be seen from Table 10, a high p-values indicates that the distribution performs well at a 95% confidence interval. For the non-parametric approach, the p-value obtained for both cryptocurrencies are very small. Hence, the null hypothesis where the expected violations are equivalent to the actual violations in the series is rejected. However, the normal distribution in parametric approach has high p-values which means that normal distribution performs better than age-weighted historical simulation method.

Independence Test

Independence test is conducted to test the independence of VaR violations as according to Christoffersen (2004), the violations in a data series need to be independent.

Table 11: Independence test for the 95%-VaR

	Bitcoin	Ethereum
Age-weighted Historical Simulation		
LR_{IND}	5.9994	4.6706
p-value	0.0143	0.0307
Conclusion	Reject	Reject
Normal distribution		
LR_{IND}	1.9507	0.0707
p-value	0.1625	0.7903
Conclusion	Fail to reject	Fail to reject

A similar result are obtained from the independence test. Under the age-weighted historical simulation method, the null hypothesis is rejected, while under the normal distribution, the null hypothesis is not rejected.

Conditional Coverage Test

The joint test of unconditional coverage test and independence test is conducted, and the result is presented in Table 12.

Table 12: Conditional coverage test for 95%-VaR

	Bitcoin	Ethereum
Age-weighted Historical Simulation		
LR_{CC}	23.6818	19.1394
p-value	0.0000072	0.00006981
Conclusion	Reject	Reject
Normal distribution		
LR_{CC}	2.1390	0.1020
p-value	0.3432	0.9502
Conclusion	Fail to reject	Fail to reject

For these two cryptocurrencies, the p-value for conditional coverage test by using normal distribution is greater than the critical level of $\alpha = 0.05$. Therefore, it can be concluded that normal distribution VaR models for Bitcoin and Ethereum have correct exceedances and are independent.

CONCLUSION

When making financial decisions, it is important to understand the risk probability. Numerous strategies have been proposed; however, they appear to have relevance only when the normality statement is considered to be true. Extreme value theory (EVT), in contrast, offers a better approach for comprehending tail returns. The main focus of this study is the downside risk of extreme returns of Bitcoin and Ethereum by using EVT. In this study, the maximum extreme returns of Bitcoin and Ethereum are estimated using the block maxima (BM) and peaks over threshold (POT) approach. The negative daily log-return of each cryptocurrency are fitted into the Generalized Extreme Value (GEV) Distribution and Generalized Pareto Distribution (GP) models. Suitable representation of the perceived data as well as sufficient accordance with the underlying theories are shown by the findings of this study. Therefore, investors who are considering cryptocurrencies in their investment portfolio can use this study as a benchmark for decision-making as this study gives some useful information about the maximum loss and VaR of Bitcoin and Ethereum. In this study, the scope of study only involves two types of cryptocurrencies which are Bitcoin and Ethereum. Besides that, there are only two methods which are chosen to estimate the VaR of each cryptocurrency. Hence, it is suggested that the scope of study be widened by involving much more cryptocurrencies like Tether (USDT), Binance Coin (BNB), USD Coin (USDC), and Dogecoin (DOGE) since every cryptocurrency has a different level of risk. It is also suggested to include different types of VaR models such as historical simulation and Monte-Carlo simulation for non-parametric approaches while Exponentially Weighted Moving Average (EWMA), Student's t-distribution and Variance Gamma (VG) distribution are used for parametric approaches.

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