Menemui Matematik (Discovering Mathematics)
Vol. 4, No. 1: 53-59 (2022)

Comparing O'Brien Test Using Mean, Median, Symmetric and Asymmetric Trimmed Mean

M. Kamal[1], N. M. Ali[2,*], N. A. A. Rahmin[2] and N. Ali[2]
[1]*Centre of Foundation Studies for Agricultural Science, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor*
[2]*Department of Mathematics and Statistics, Faculty of Science, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor,*
[1]munirahkamal@upm.edu.my, [2]nazihanma@upm.edu.my

ABSTRACT

The assumption of homogeneity of variances can be tested with O'Brien procedure since it is compatible with standard analysis of variance (ANOVA). The test is very sensitive to non-normality which lead to find the alternative technique by replacing the original test with the robust measure of location. This study will investigate the behaviour of O'Brien test on robustness by using the usual variances on mean, median, symmetric trimmed and asymmetric trimmed mean under normal distribution and skewed normal distributions. The results show that O'Brien test is robust when data is distributed at normal and skewed normal under certain simulation study condition.

Keywords: ANOVA, O'Brien, trimmed, symmetric, asymmetric

INTRODUCTION

The assumption of homogeneity of variance is used to test whether the variances for the groups or populations are equal or not. The O'Brien test is used to transform the original values so that the transformed values reflect the variation of the original values. Hence, a standard of analysis of variance (ANOVA) will test the homogeneity of variance assumption by using the transformed values.

According to Abdi (2007), O'Brien test using the real data example gives the satisfactory results on testing the homogeneity of variance by computing the original scores to transformed scores on the median instead of the mean. Keselman et al. (2008), Lee et al. (2010) and Othman et al. (2012) had modified the procedure suggested by O'Brien (1981) by developed the method that based on robust estimators to overcome the bias effects of variance heterogeneity and non-normality. Then, they used the modified data into the estimates in ANOVA F-test.

The main objective of this study is to find the alternative method on testing the assumption of homogeneity of variances by controlling Type I error. In this study, we examined and compared these alternative procedures on the O'Brien test by replacing the mean by using robust estimators, symmetric trimmed mean and asymmetric trimmed mean. The trimming methods conducted on O'Brien test, so that the test will produce good Type I error when dealing with non-normal distribution. Hence, results from this study will give the researcher the alternative procedures of O'Brien test in terms of robustness when dealing with outliers or non-normality.

O'Brien test is proposed by O'Brien (1979) to test the assumption of homogeneity of variance,

$$H_0 : \sigma_1{}^2 = \sigma_2{}^2 = \ldots = \sigma_j{}^2$$
$$H_1 : \sigma_i^2 \neq \sigma_j^2 \quad \text{for at least one pair } (i, j).$$

where *j* is the number of independent groups. A standard application of this O'Brien test is to replace y_{ij} the original scores by the transformed scores denoted as $y_{ij}(w)$,

$$y_{ij}(w) = \frac{(n_j - w - 1)n_i(y_{ij} - \bar{y}.)^2 - ws_i^2(n_i - 1)}{(n_i - 1)(n_i - 2)} \tag{1}$$

where n_i is the sample size of the i^{th} subgroup, w is a parameter with range $0 \le w \le 1$, $\bar{y}. = \frac{\sum_{j=1}^{n_i} y_{ij}}{n_i}$ and $s_i^2 = \frac{\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_{i.})^2}{(n_i - 1)}$.

O'Brien (1981) has suggested to set $w = 0.5$ as a default to balance the expected moderate departure from kurtosis that equal to zero. By setting $w = 0.5$, equation (1) become

$$y_{ij}(0.5) = \frac{(n_j - 1.5)n_i(y_{ij} - \bar{y}.)^2 - 0.5s_i^2(n_i - 1)}{(n_i - 1)(n_i - 2)} \tag{2}$$

By using this new score, the standard analysis of variance will be calculated by using equation

$$F_{O'Brien} = \frac{\sum_{j=1}^{t} n_i(\bar{r}_{i.} - \bar{r}_{..})^2 / (t-1)}{\sum_{i=1}^{t}\sum_{j=1}^{n_i}(\bar{r}_{ij} - \bar{r}_{i.}) / \sum_{i=1}^{t}(n_i - 1)} \tag{3}$$

where t is the number of the i^{th} subgroup, $\bar{r}_{i.} = \frac{\sum_{j=1}^{n_i} r_{ij}}{n_i}$ and $\bar{r}_{..} = \frac{\sum_{i=1}^{t}\sum_{j=1}^{n_i}\bar{r}_{ij}}{n_i}$ for $i = 1, 2, \ldots, j$ and $j = 1, 2, \ldots, n$.

Two different trimming method has been used in the sense that, for the fixed trimmed mean method, data is trimmed equally on both sides of the distribution. While for asymmetric trimmed mean, data is trimmed based on the skewed side of the distribution. For symmetric trimmed mean, let $X_{(1)j} \le X_{(2)j} \le \ldots \le X_{nj}$ represent the ordered observations associated with the j^{th} group. The j^{th} group trimmed mean is given by

$$\bar{X}_{tj} = \frac{1}{(1-2g)n_j}\left[\sum_{i=k+1}^{n_j - k} X_{(i)j} + r(X_{(k)j} + X_{(n_j - k + 1)j})\right] \tag{4}$$

where g represents the proportion of observations that are to be trimmed in each tail of the distribution, n_j is the sample size of each group and $k = [gn_j] + 1$ where $[gn_j]$ is the largest integer $\le gn_j$ and $r = k - gn_j$.

For asymmetric trimmed mean,

$$\bar{X}_{tj} = \frac{1}{(1-2\gamma)n_j}\left[\sum_{i=k+1}^{n_j-k_j} X_{(i)j} + (X_{(k)j} - \gamma n_j)(X_{(k)j} + X_{n_j-k_j-1})\right] \tag{5}$$

where γ is a proportion that has been trimmed from each tail, n_j is the sample size of each group and $k_j = [\gamma n_j] + 1$ where $[\gamma g n_j]$ is the largest integer $\leq g n_j$.

SIMULATION STUDY

A total of 180 O'Brien procedures were examined and investigated under all simulation condition as in Table 1.

Table 1: Description of the O'Brien test used in the new simulation

Designation	Description	Cases
M1	$X_{ij} \to R_{ij}$: use group means and variances from X_{ij}. R_{ij} : apply usual ANOVA F-test at α : 0.01; 0.05.	2
M2	$X_{ij} \to R_{ij}$: use group medians and variances based on the medians from X_{ij}. R_{ij} : apply usual ANOVA F-test at α : 0.01; 0.05.	2
S1	$X_{ij} \to R_{ij}$: use group symmetric trimmed means and usual variances from X_{ij}. X_{ij} symmetrically trimmed at tail proportions: 0.05, 0.10, 0.15, 0.20. R_{ij} : apply usual ANOVA F-test at α : 0.01; 0.05.	8
AS1	$X_{ij} \to R_{ij}$: use group asymmetric trimmed means and usual variances from X_{ij}. X_{ij} asymmetrically trimmed at tail proportions: 0.05, 0.10, 0.15, 0.20. R_{ij} : apply usual ANOVA F-test at α : 0.01; 0.05.	8

(M1) Variants are designated M1. O'Brien transformation based upon group means and variances used with the usual F-test at $\alpha : 0.01$ and 0.05.

(M2) Variants are designated M2. The O'Brien transformation based upon group medians and variances based on the medians used with the usual F-test at $\alpha : 0.01$ and 0.05.

(S1) Variants are designated S1. The O'Brien transformation based upon symmetric trimmed means and usual variances of X_{ij}. These trimmed means were calculated at tail proportions 0:05, 0:10, 0:15 and 0:20. The transformed values, were used with the usual F-test at $\alpha : 0.01$ and 0:05. Because there are four symmetrical trimming percentages, there are four variants with this designation. For example, variants $S_1 5$ signifies transformation of with 5% symmetric trimmed mean.

(AS1) Variants are designated AS1. The O'Brien transformation based upon asymmetric trimmed means and usual variances of X_{ij}. These trimmed means were calculated at tail proportions 0.05, 0.10, 0.15 and 0.20. The transformed values, r_{ij} were used with the usual F-test at $\alpha: 0.01$ and 0.05. Because there are four trimming percentages, there are four variants with this designation. For example, variants $AS_1 5$ signifies transformation of with 5% asymmetric trimmed mean.

Several conditions were taken into account in order to test the homogeneity of variances. For each 18 cases in Table 1, test was conducted based on the condition of sample size, type of distribution and percentage of total trimming, will give the total of 180 procedure.

Sample size

In order to test for Type I error, balanced and unbalanced group sizes were assigned to the case for three groups. The total number of sample size was fixed about 120 samples. Three conditions of sample size were investigated that are equal, moderately unequal and extremely unequal. The values for each condition are presented in the Table 2.

Table 2: Sample size condition

Sample size	Condition
40, 40, 40	Equal
35, 40, 45	Moderately unequal
30, 40, 50	Extremely unequal

Type of population distribution

Three types of distribution representing different levels of skewness was used using *g* and *h* distributions (Hoaglin, 1985). The *g* and *h* distributions are modified from standard normal distribution with constant *g* controlling the value of skewness and *h* controlling the value of kurtosis. Different values of (*g*, *h*) has been used in this study were (0, 0), (0.5, 0) and (0.5, 0.5). Table 3 summarize the shape of *g* and *h* distributions (Wilcox, 1997).

Table 3: Properties of *g* and *h* distributions

g	*h*	Skewness	Kurtosis	Shape
0.0	0.0	0.0	3.0	Normal
0.5	0.0	1.81	9.7	Skewed normal-tailed
0.5	0.5	120.1	18393.6	Skewed heavy-tailed

Percentage of total trimming

Four values of total trimming namely 5%, 10%,15% and 20% were examined when data were symmetrically and asymmetrically trimmed to obtain the values used in the transformation of X_{ij} data when trimming is carried out O'Brien transformed values r_{ij}.

For each condition, 10000 replications were conducted using R programming and the performances of robustness and power is tested at $\alpha: 0.01$ and $\alpha: 0.05$.

RESULTS AND DISCUSSION

The test is said to be robust when the value of Type I error is bounded in interval $0.5\alpha < \alpha < 1.5\alpha$. The evaluation scales are followed Bradley's liberal criterion of robustness (Bradley, 1978). As stated, we investigated the value of Type I error for $\alpha : 0.01$ and $\alpha : 0.05$, the summary of Bradley's liberal criterion of robustness is tabulated in Table 4.

Table 3: Properties of *g* and *h* distributions

Type I Error Rate	$\alpha = 0.01$	$\alpha = 0.05$	Criterion of Robustness
Less than 0.5α	$\alpha < 0.005$	$\alpha < 0.025$	Conservative
Between $0.5\alpha < \alpha < 1.5\alpha$	$0.005 < \alpha < 0.015$	$0.025 < \alpha < 0.075$	Robust
Greater than 1.5α	$\alpha > 0.015$	$\alpha > 0.075$	Liberal

Table 5 and Table 6 reported the Type 1 error rates of all compared test under all condition.

Table 5: O'Brien test by using mean-based and median-based, with variance, $\sigma_i^2 = (1,\ 1,\ 1)$

α	Sample size n_i	Normal		Skewed Normal-tailed		Skewed Heavy-tailed	
		M1	M2	M1	M2	M1	M2
0.01	(40,40,40)	**0.0087**	**0.0063**	**0.0076**	0.0030	0.0016	0.0006
	(35,40,45)	**0.0079**	**0.0061**	**0.0085**	0.0031	0.0014	0.0006
	(30,40,50)	**0.0088**	**0.0070**	**0.0090**	0.0034	0.0027	0.0011
0.05	(40,40,40)	**0.0476**	**0.0415**	**0.0439**	0.0214	0.0146	0.0093
	(35,40,45)	**0.0471**	**0.0409**	**0.0416**	0.0215	0.0179	0.0113
	(30,40,50)	**0.0466**	**0.0401**	**0.0437**	0.0224	0.0189	0.0129

Note: *Bold values indicate that Type I error is robust based on Bradley criterion.

Table 6: Symmetric and asymmetric trimming O'Brien test with variance, $\sigma_i^2 = (1,\ 1,\ 1)$

α	g	Sample size n_i	Normal		Skewed Normal-tailed		Skewed Heavy-tailed	
			S1	AS1	S1	AS1	S1	AS1
0.01	0.05	(40,40,40)	**0.0080**	**0.0079**	**0.0051**	0.0047	0.0009	0.0008
		(35,40,45)	**0.0074**	**0.0074**	**0.0053**	**0.0052**	0.0011	0.0011
		(30,40,50)	**0.0082**	**0.0079**	**0.0059**	**0.0056**	0.0015	0.0013
	0.10	(40,40,40)	**0.0076**	**0.0076**	0.0043	0.0042	0.0007	0.0006
		(35,40,45)	**0.0068**	**0.0068**	0.0041	0.0040	0.0008	0.0007
		(30,40,50)	**0.0073**	**0.0072**	0.0046	0.0041	0.0013	0.0012
	0.15	(40,40,40)	**0.0074**	**0.0073**	0.0039	0.0037	0.0006	0.0006

		(35,40,45)	**0.0066**	**0.0066**	0.0036	0.0035	0.0006	0.0005
		(30,40,50)	**0.0070**	**0.0070**	0.0039	0.0037	0.0011	0.0011
	0.20	(40,40,40)	**0.0068**	**0.0068**	0.0036	0.0034	0.0006	0.0006
		(35,40,45)	**0.0065**	**0.0063**	0.0033	0.0031	0.0006	0.0006
		(30,40,50)	**0.0080**	**0.0079**	0.0035	0.0033	0.0010	0.0010
0.05	0.05	(40,40,40)	**0.0452**	**0.0451**	**0.0301**	**0.0275**	0.0100	0.0094
		(35,40,45)	**0.0448**	**0.0448**	**0.0292**	**0.0281**	0.0123	0.0121
		(30,40,50)	**0.0438**	**0.0436**	**0.0313**	**0.0298**	0.0143	0.0139
	0.10	(40,40,40)	**0.0444**	**0.0440**	0.0238	0.0236	0.0090	0.0090
		(35,40,45)	**0.0439**	**0.0437**	0.0247	0.0242	0.0112	0.0110
		(30,40,50)	**0.0426**	**0.0426**	**0.0269**	**0.0257**	0.0132	0.0130
	0.15	(40,40,40)	**0.0432**	**0.0428**	0.0228	0.0220	0.0089	0.0089
		(35,40,45)	**0.0430**	**0.0429**	0.0225	0.0220	0.0107	0.0108
		(30,40,50)	**0.0420**	**0.0420**	0.0243	0.0239	0.0128	0.0127
	0.20	(40,40,40)	**0.0417**	**0.0416**	0.0218	0.0214	0.0090	0.0091
		(35,40,45)	**0.0427**	**0.0424**	0.0214	0.0210	0.0109	0.0110
		(30,40,50)	**0.0415**	**0.0415**	0.0231	0.0224	0.0129	0.0129

Note: *Bold values indicate that Type I error is robust based on Bradley criterion.

The results of Table 5 and 6 reveal that the variants that give the best result for Type I error and can be summarize as:
For $\alpha = 0.01$;
(1) Designation M1. The O'Brien transformation based upon group means and variances with the usual F-test under normal distribution for *n* = 40; 40; 40 and *n* = 30; 40; 50; and under skewed normal-tailed distribution for *n* = 35; 40; 45.
(2) Designation $S_1$5. The O'Brien transformation of X_{ij} with 5% symmetric trimmed mean and usual variances under normal distribution for *n* = 40; 40; 40 and *n* = 30; 40; 50.
(3) Designation $S_1$20. The O'Brien transformation of X_{ij} with 20% symmetric trimmed mean and usual variances under normal distribution for *n* = 30; 40; 50.

For $\alpha = 0.05$:
(4) Designation M1. The O'Brien transformation based upon group means and variances with the usual F-test under normal distribution for *n* = 40; 40; 40, *n* = 35; 40; 45 and *n* = 30; 40; 50.
(5) Designation $S_1$5 and $AS_1$5. The O'Brien transformation of X_{ij} with 5% symmetric trimmed mean and usual variances under normal distribution for *n* = 40; 40; 40.

CONCLUSION

In term of robustness, all tests under normal distribution showed that Type I error is robust under all simulation conditions of total sample size and the total amount of trimming. However, for skewed normal tailed distribution, only certain conditions are filled the robustness criterion while the rest are conservative for skewed heavy-tailed distribution. Therefore, it can be said that the F-test performs well when both normality and homogeneity of variance are true, while by using the symmetric and asymmetric trimmed mean, O'Brien procedure give the robust result for skewed normal distribution with 5% percentage of trimming.

REFERENCES

Abdi, H. (2007). O'Brien test for homogeneity of variance. Encyclopaedia of Measurement and Statistics, 2:701–704.

Bradley, J. V. (1978). Robustness? British Journal of Mathematical and Statistical Psychology, 31(2):144–152.

Hoaglin, D. C. (1985). Summarizing shape numerically: The g- and h- distributions. Journal of the American Statistical Association, pages 461–513.

Keselman, H., Wilcox, R. R., Algina, J., Othman, A. R., and Fradette, K. (2008). A comparative study of robust tests for spread: asymmetric trimming strategies. British Journal of Mathematical and Statistical Psychology, 61(2):235–253.

Lee, H. B., Katz, G. S., and Restori, A. F. (2010). A monte carlo study of seven homogeneity of variance tests. Journal of Mathematics and Statistics, 6(3):359.

O'Brien, R. G. (1979). A general ANOVA method for robust tests of additive models for variances. Journal of the American Statistical Association, 74(368):877–880.

O'Brien, R. G. (1981). Quantitative methods in psychology. Psychological Bulletin, 89(3):570–574.

Othman, A. R., Keselman, H., Wilcox, R. R., Algina, J., et al. (2012). Robust modifications of the Levene and O'Brien tests for spread. Journal of Modern Applied Statistical Methods, 11(1):5.

Wilcox, R. R. (1997). Introduction to robust estimation and Hypothesis testing. San Diego, CA: Academic Press.