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Nonexistence of global solutions for the Timoshenko equation with degenerate damping

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ABSTRACT
In this work, we consider a Timoshenko equations with degenerate damping term. We prove the nonexistence of global solutions with arbitrary positive initial energy. This result is extensions of earlier results.

Keywords: Nonexistence of solutions, Timoshenko equation, Degenerate damping

INTRODUCTION AND PRELIMINARIES

In this work, we focus on the nonexistence of solution in finite time for the following problem

$$\begin{cases} u_{tt}+\Delta^2 u+M\left(\|\nabla u\|^2\right)(-\Delta u)+|u|^{\vartheta} j'(u_t)=|u|^{q-1}u, & (x,t)\in\Omega\times(0,T), \\ u(x,t)=\dfrac{\partial}{\partial\nu}u(x,t)=0, & (x,t)\in\partial\Omega\times(0,T), \\ u(x,0)=u_0(x), \quad u_t(x,0)=u_1(x), & x\in\Omega, \end{cases} \tag{1}$$

where $1<q<\infty$, $\vartheta\geq 0$ and Ω is a bounded domain in R^n with a smooth boundary $\partial\Omega$. $j(s)$ is a continuous, convex, real-valued function defined on R and $j'(s)$ is derivative of $j(s)$ (Barbu, 1976). $M(s)$ is a function for $s\geq 0$ satisfying

$$M(s)=1+s^{\gamma}, \quad \gamma>1.$$

In (Woinowsky-Krieger, 1976), Woinowsky-Krieger firstly proposed the so called Beam equation or Timoshenko equation model

$$\frac{\partial^2 u}{\partial t^2}+\frac{\partial^4 u}{\partial x^4}+\left(\alpha_1+\beta_1\int_0^L\left|\frac{\partial u}{\partial t}\right|d\tau\right)\left(-\frac{\partial^2 u}{\partial x^2}\right)+g\left(\frac{\partial u}{\partial t}\right)=0, \tag{2}$$

for $g=0$, where $\alpha_1,\beta_1>0$ are constants and $u(x,t)$ is the deflection of the point x of the beam at the time t.

When $\vartheta=0$ and $j'(u_t)=|u_t|^{p-2}u_t$, (1) become the following equation

$$u_{tt}+\Delta^2 u+M\left(\|\nabla u\|^2\right)(-\Delta u)+|u_t|^{p-2}u_t=|u|^{q-2}u. \tag{3}$$

In [(Esquivel-Avila, 2011), (Esquivel-Avila, 2013)], Esquivel-Avila showed the global attractor, convergence and unboundedness of solutions for the equation (3). In (Pişkin, 2005), Pişkin studied the existence, decay estimates of solutions and blow up of solutions with negative initial energy for this equation. Then, Pişkin and Irkıl (Pişkin and Irkıl, 2016) proved blow up of solutions with positive initial energy for this equation. Recently, Periara et al. (2019) and Pişkin and Yüksekkaya (2018) studied this equation in case of $p=2$. Periara et al. proved existence of the global solutions through the Faedo-Galerkin approximations and studied the asymptotic behavior by using the

Nakao method. Pişkin and Yüksekkaya proved the blow up of solutions with positive and negative initial energy.

When problem (1) without fourth order term $\Delta^2 u$ and $M \equiv 1$, problem (1) can be reduced to the classical problem

$$u_{tt} - \Delta u + |u|^{\vartheta} j'(u_t) = |u|^{q-1} u. \quad (4)$$

It is worth noting that problem (4) has been widely investigated by numerous authors and several results concerning nonexistence, asymptotic stability, existence and uniquness of solutions under consideration of relation between exponent of the source term and exponent of damping terms (see references in Barbu et. al (2007), Barbu et. al (2005), Barbu et. al (2005), Hu and Zhang (2007), Xiao and Shubin (2019)).

It should be noted here that Eq.(4) with Kirchhoff-type and fourth order term $\Delta^2 u$ is quite different from those types of mentioned research work above. As far as the author knows, our model is a generalization of what (4), but our model is quite difficult and particularly interesting due to Kirchhoff-type and $\Delta^2 u$ term and has not been discussed before. There is not other result on about solutions of Timoshenko problems with degenerate damping terms.

The object of this paper is to state and study nonexistence of weak solutions (1) under the sufficent conditions on the arbitrary positive initial energy in the main result by using an idea used in (Xiao and Shubin, 2019). Now, we present some preliminary material which will be helpful in the proof of our result. In order to simplify the notations, we denote

$$\|\cdot\| = \|\cdot\|_{L^p(\Omega)}, \quad \|\cdot\| = \|\cdot\|_{L^2(\Omega)}.$$

As in Barbu et. al (2007), Barbu et. al (2005), Hu and Zhang (2007), Xiao and Shubin (2019), we give the following assumptions.

Assumption (A) :

- $\vartheta, p, q \geq 0.$ In addition, $\vartheta \leq \frac{n}{n-2}$, $q \leq \frac{n}{n-2}$, if $n \geq 3$;
- $j'(\alpha)$ is a single valued and $|j'(\alpha)| \leq c_0 |a|^p$;
- $(j'(\alpha) - j'(\beta))(\alpha - \beta) \geq c_1 |\alpha - \beta|^{p+1}$,

where c_0, c_1 are positive constants for al $\alpha, \beta \in R.$

Definiton 1.1: Suppose that $u \in C\left([0,T]; H_0^1(\Omega) \cap C_w^1\left([0,T]; L^2(\Omega)\right)\right)$ which satisfies $M\left(\|\nabla u\|^2\right)\Delta u \in L^{2(\gamma+1)}\left(\Omega \times (0,T)\right)$, u_{tt} and $|u|^{\vartheta} j'(u_t) \in L^2\left(\Omega \times (0,T)\right)$ is said to be a weak solution to (1) if the following variational equality holds:

$$(u_{tt}, \varphi) + (\Delta^2 u, \varphi) + \left(\left(1 + \left(\int_\Omega |\nabla u|^2 dx\right)^{\gamma}\right)\nabla u, \nabla \varphi\right) - \left(\int_\Omega |u|^{\vartheta} j'(u_t) dx, \varphi\right) = \left(\int_\Omega |u|^{q-1} u dx, \varphi\right),$$

for any $\varphi \in L^2\left(\Omega \times (0,T)\right)$, and for all $t \in (0,T]$ and initial data $u_0(x) \in H_0^1(\Omega)$, $u_1(x) \in L^2(\Omega)$.

We introduce the energy function such that

$$E(t) = \frac{1}{2}\|u_t\|^2 + \frac{1}{2}\left(\|\nabla u\|^2 + \|\Delta u\|^2\right) + \frac{1}{2(\gamma+1)}\|\nabla u\|^{2(\gamma+1)} - \frac{1}{q+1}\|u\|_{L^{q+1}}^{q+1}.$$

Lemma 1.2: Let $E(t)$ be a energy functional of problem (1), then we have

$$\frac{d}{dt}E(t) = -\int_\Omega |u|^{\vartheta} j'(u_t) u_t dx. \quad (5)$$

Proof:
By multiplying equation (1) by u_t, integrating over Ω. Then, we get

$$\int_0^t E'(\tau)d\tau = -\int_0^t \int_\Omega |u|^\vartheta j'(u_\tau)u_\tau dxd\tau,$$

$$E(t) - E(0) = -\int_0^t \int_\Omega |u|^\vartheta j'(u_\tau)u_\tau dxd\tau \text{ for } t \geq 0.$$

NONEXISTENCE OF GLOBAL SOLUTIONS

In this section, we need the following lemma to prove our main result.
Lemma 2.1: Let ϑ, p, q, c_0 be constants, $q > \vartheta + p \geq 1$, and

$$K(L) \equiv q + 1 - \frac{c_0(\vartheta + p + 1)(\vartheta + p - 1)}{L(p+1)(q-1)},$$

$$\alpha(L) = \left[(K(L)+2)\left((K(L)-2)\lambda_1 - \frac{c_0(\vartheta + p + 1)(q - \vartheta - p)}{L(p+1)(q-1)} \right) \right]^{\frac{1}{2}}.$$

Then, there exists $L_1 > 0$ such that $\alpha(L)$ is well defined on $[L_1, +\infty)$, and the following equality

$$\frac{K(L)}{\alpha(L)} = \frac{pc_0}{(p+1)c_1} L^{\frac{1}{p}}, \tag{6}$$

holds for some constant L on $(L_1, +\infty)$.

Proof:
Let

$$\alpha_0(L) = (K(L)-2)\lambda_1 - \frac{c_0(\vartheta + p + 1)(q - \vartheta - p)}{L(p+1)(q-1)},$$

$$L_0 = \frac{c_0(\vartheta + p + 1)(\vartheta + p - 1)}{(p+1)(q-1)^2} > 0.$$

Obviously, $K(L_0) = 2$, $K(L) > 2$ for any $L > L_0$, and

$$\alpha(L) \to -\frac{(q - \vartheta - p)(q-1)}{\vartheta + p - 1} < 0, \text{ as } L \to L_0. \tag{7}$$

On the other hand, noting that

$$\alpha(L) \to (q-1)\lambda_1 > 0, \text{ as } L \to +\infty,$$

which combined with (7) and the fact that $\alpha_0(L)$ is increasing on $(0, +\infty)$, yields that $\alpha_0(L) = 0$ possesses an unique solution L_1 on $(L_0, +\infty)$. Noting that $K(L) + 2 > 0$ for any $L > L_0$, then $\alpha(L) > 0$ for any $L > L_1$, which means $\alpha(L)$ is well defined on $[L_1, +\infty)$.

At last, by a straightforward calculation, we can obtain that

$$\lim_{L \to L_1} \frac{K(L)}{\alpha(L)} - \frac{pc_0}{(p+1)c_1} L^{\frac{1}{p}} = +\infty, \quad \lim_{L \to +\infty} \frac{K(L)}{\alpha(L)} - \frac{pc_0}{(p+1)c_1} L^{\frac{1}{p}} = -\infty.$$

Then, by means of the mean value theorem for continuous functions the equation (6) has a root L on $(L_1, +\infty)$. Thus, the proof of the lemma is complete.

Our main result reads as follows:

Theorem 2.2: Assume that (A) and $q > \vartheta + p \geq 1$ hold. Let $u_0(x) \in H_0^1(\Omega)$, $u_1(x) \in L^2(\Omega)$ and u is a local solution of the problem (1). Also, if $E(0) > 0$ and

$$(u_0, u_1) > L^{\frac{1}{p}} \frac{pc_0}{(p+1)c_1} E(0), \tag{8}$$

then the weak solutions to (1) blow up in a finite time, where L is the root of the equation (6).
Proof: We assume that u is a global solution and we arrive at a contradiction. For this aim

$$F(t) = \|u\|^2.$$

Direct derivation of $F(t)$, we have

$$F'(t) = 2(u, u_t).$$

Noting that $u_{tt} \in H^{-1}(\Omega)$ for all $0 \leq t < T$, the standard approximation argument shows that $F''(t)$ exists and

$$\begin{aligned} F''(t) &= 2(u, u_{tt}) + 2\|u_t\|^2 \\ &= 2\|u_t\|^2 - 2\|\Delta u\|^2 - 2\|\nabla u\|^2 - 2\|\nabla u\|^{2(\gamma+1)} + 2\|u\|_{L^{q+1}}^{q+1} - 2\int_\Omega |u|^\vartheta j'(u_t) u dx. \end{aligned}$$

Set

$$M(t) = F'(t) - \eta E(t), \qquad \eta = L^{\frac{1}{p}} \frac{2pc_0}{(p+1)c_1}. \tag{9}$$

Step 1: We prove the fact that

$$M(t) \geq M(0) e^{\alpha(L)t} > 0, \qquad \lim_{t \to +\infty} M(t) = +\infty. \tag{10}$$

It follows from (9), the energy equality (5) and assumption (A) that

$$\begin{aligned} \frac{d}{dt} M(t) &= F''(t) - \eta E'(t) \\ &= 2\|u_t\|^2 - 2\|\Delta u\|^2 - 2\|\nabla u\|^2 - 2\|\nabla u\|^{2(\gamma+1)} + 2\|u\|_{L^{q+1}}^{q+1} \\ &\quad -2\int_\Omega |u|^\vartheta j'(u_t) u dx + \eta \int_\Omega |u|^\vartheta j'(u_t) u_t dx \\ &\geq 2\|u_t\|^2 - 2\|\Delta u\|^2 - 2\|\nabla u\|^2 - 2\|\nabla u\|^{2(\gamma+1)} + 2\|u\|_{L^{q+1}}^{q+1} \\ &\quad -2c_0 \int_\Omega |u|^{\vartheta+1} |u_t|^p dx + \eta c_1 \int_\Omega |u|^\vartheta |u_t|^{p+1} dx. \end{aligned} \tag{11}$$

By means of the Hölder's inequality and Young's inequality, we obtain

$$\begin{aligned} \int_\Omega |u|^{\vartheta+1} |u_t|^p dx &= \int_\Omega |u|^{\vartheta+1-\frac{\vartheta p}{p+1}} |u|^{\frac{\vartheta p}{p+1}} |u_t|^p dx \\ &\leq \left(\int_\Omega |u|^{\vartheta+p+1} dx\right)^{\frac{1}{p+1}} \left(\int_\Omega |u|^\vartheta |u_t|^{p+1} dx\right)^{\frac{p}{p+1}} \\ &\leq \frac{1}{p+1} \delta^{p+1} \int_\Omega |u|^{\vartheta+p+1} dx + \frac{p}{p+1} \delta^{-\frac{p+1}{p}} \int_\Omega |u|^\vartheta |u_t|^{p+1} dx, \end{aligned} \tag{12}$$

for $\delta > 0$. Substituting (12) into (11), (11) becomes such that

$$\frac{d}{dt}M(t) \geq 2\|u_t\|^2 - 2\|\Delta u\|^2 - 2\|\nabla u\|^2 - 2\|\nabla u\|^{2(\gamma+1)} + 2\|u\|_{L^{q+1}}^{q+1}$$
$$- \frac{2c_0}{p+1}\delta^{p+1}\int_\Omega |u|^{\vartheta+p+1}dx + \left(\eta c_1 - \frac{2c_0 p}{p+1}\delta^{-\frac{p+1}{p}}\right)\int_\Omega |u|^\vartheta |u_t|^{p+1}dx. \tag{13}$$

Observe that the function

$$g(y) = \frac{a^y}{y}, \quad a \geq 0, \quad a \neq 1, \quad y > 0 \tag{14}$$

is convex and note that for $q > \vartheta + p \geq 1$,

$$\frac{q-\vartheta-p}{q-1} + \frac{p+\vartheta-1}{q-1} = 1,$$
$$p+\vartheta+1 = 2\frac{q-\vartheta-p}{q-1} + (q+1)\frac{p+\vartheta-1}{q-1}.$$

By the properties of convex functions, we have

$$\int_\Omega \frac{|u|^{\vartheta+p+1}}{\vartheta+p+1}dx \leq \frac{q-\vartheta-p}{2(q-1)}\int_\Omega |u|^2 dx + \frac{p+\vartheta-1}{(q+1)(q-1)}\int_\Omega |u|^{q+1}dx. \tag{15}$$

From (13) and (15), we can obtain

$$\frac{d}{dt}M(t) \geq 2\|u_t\|^2 - 2\|\Delta u\|^2 - 2\|\nabla u\|^2 - 2\|\nabla u\|^{2(\gamma+1)} + 2\|u\|_{L^{q+1}}^{q+1}$$
$$- \frac{2c_0(p+\vartheta+1)(p+\vartheta-1)}{L(p+1)(q^2-1)}\|u\|_{L^{q+1}}^{q+1}$$
$$- \frac{c_0(p+\vartheta+1)(q-\vartheta-p)}{L(p+1)(q-1)}\|u\|^2$$
$$\geq 2\|u_t\|^2 - 2\|\Delta u\|^2 - 2\|\nabla u\|^2 - 2\|\nabla u\|^{2(\gamma+1)}$$
$$- \frac{2K(L)}{q+1}\|u\|_{L^{q+1}}^{q+1}$$
$$- \frac{c_0(p+\vartheta+1)(q-\vartheta-p)}{L(p+1)(q-1)}\|u\|^2.$$

Recalling $E(t)$, Poincare's inequality, and $K(L) > 2(\gamma+1)$, it holds

$$\frac{d}{dt}M(t) \geq (2+K(L))\|u_t\|^2 + (K(L)-2)\|\Delta u\|^2 + (K(L)-2)\|\nabla u\|^2$$
$$+ \left(\frac{K(L)}{\gamma+1} - 2\right)\|\nabla u\|^{2(\gamma+1)} - 2K(L)E(t)$$
$$- \frac{c_0(p+\vartheta+1)(q-\vartheta-p)}{L(p+1)(q-1)}\|u\|^2,$$

$$\frac{d}{dt}M(T)\geq\left(2+K(L)\right)\|u_t\|^2-2K(L)E(t)$$
$$\left(\left(K(L)-2\right)\lambda_1-\frac{c_0\left(p+\vartheta+1\right)\left(q-\vartheta-p\right)}{L\left(p+1\right)\left(q-1\right)}\right)\|u\|^2$$
$$\geq 2\sqrt{\left(2+K(L)\right)\alpha_0(L)}\left(u,u_t\right)-2K(L)E(t)$$
$$=\alpha(M)\left(2\left(u,u_t\right)-\frac{2K(L)}{\alpha(L)}E(t)\right).$$

It follows from $\frac{K(L)}{\alpha(L)}=\frac{pc_0}{(p+1)c_1}L^{\frac{1}{p}}$ that

$$\frac{d}{dt}M(t)\geq\alpha(L)M(t).$$

The condition (8) guarantees $M(0)>0.$ Thus, it yields (10).

Step 2: We assume that u is a global solution and we arrive at a contradiction. From the expression (9) that

$$\begin{aligned}F(t)&=\|u_0\|^2+\int_0^t F'(\tau)\,d\tau\\&>\|u_0\|^2+\int_0^t M(\tau)\,d\tau\\&>\|u_0\|^2+\frac{M(0)}{\alpha(L)}\left(e^{\alpha(L)t-1}\right).\end{aligned}\tag{16}$$

Note that

$$|u|^{p+\vartheta+1}-|u_0|^{p+\vartheta+1}=\int_0^t\frac{d}{d\tau}|u|^{p+\vartheta+1}\,d\tau,$$

then by a straight forward calculation, we have

$$\begin{aligned}\int_\Omega|u|^{p+\vartheta+1}dx&=\int_\Omega|u_0|^{p+\vartheta+1}dx+\int_\Omega\int_0^t\frac{d}{d\tau}|u|^{p+\vartheta+1}d\tau dx\\&\leq\int_\Omega|u_0|^{p+\vartheta+1}dx+\left(p+\vartheta+1\right)\int_0^t\int_\Omega|u|^{\vartheta+p}|u_\tau|\,dxd\tau.\end{aligned}\tag{17}$$

Now, we estimate $\int_0^t\int_\Omega|u|^{\vartheta+p}|u_\tau|\,dxd\tau$ term in (17). By using the Hölder's inequality, assumption (A) and the Young's inequality, we obtain

$$\begin{aligned}\int_0^t\int_\Omega|u|^{\vartheta+p}|u_\tau|\,dxd\tau&=\int_0^t\int_\Omega|u|^{\vartheta+p-\frac{\vartheta}{p+1}}|u|^{\frac{\vartheta}{p+1}}|u_\tau|\,dxd\tau\\&\leq\int_0^t\left(\int_\Omega|u|^{p+\vartheta+1}dx\right)^{\frac{p}{p+1}}\left(\int_\Omega|u|^{\vartheta}|u_\tau|^{p+1}dx\right)^{\frac{1}{p+1}}d\tau\\&\leq\frac{1}{c_1}\int_0^t\|u\|_{L^{\vartheta+p+1}}^{\frac{p(p+\vartheta+1)}{p+1}}\int_0^t\left(\int_\Omega|u|^{\vartheta}\,j'(u_\tau)u_\tau dx\right)^{\frac{1}{p+1}}d\tau\\&\leq\frac{1}{c_1}\|u\|_{L^\infty\left(0,t;L^{\vartheta+p+1}\right)}^{\frac{p(p+\vartheta+1)}{p+1}}\int_0^t\left(\int_\Omega|u|^{\vartheta}\,j'(u_\tau)u_\tau dx\right)^{\frac{1}{p+1}}d\tau\\&\leq\frac{1}{c_1}t^{-\frac{p}{p+1}}\|u\|_{L^\infty\left(0,t;L^{\vartheta+p+1}\right)}^{\frac{p(p+\vartheta+1)}{p+1}}\left(\int_0^t\int_\Omega|u|^{\vartheta}\,j'(u_\tau)u_\tau dxd\tau\right)^{\frac{1}{p+1}}\end{aligned}$$

for $\varepsilon > 0.$ Since $E(t) \geq 0,$ from the energy identity (5) we get $\int_0^t \int_\Omega |u|^\vartheta j'(u_\tau) u_\tau dx d\tau \leq E(0),$ then it follows that

$$\begin{aligned} \int_0^t \int_\Omega |u|^{\vartheta+p} |u_\tau| dx d\tau &\leq \varepsilon \|u\|_{L^\infty(0,t;L^{\vartheta+p+1})}^{p+\vartheta+1} + C_\varepsilon t^p \int_0^t \int_\Omega |u|^\vartheta j'(u_\tau) u_\tau dx d\tau \\ &\leq \varepsilon \|u\|_{L^\infty(0,t;L^{\vartheta+p+1})}^{p+\vartheta+1} + C_\varepsilon t^p E(0), \end{aligned} \tag{18}$$

for $0 < \varepsilon < \dfrac{1}{p+\vartheta+1}.$ Substituting (18) into (17), it follows that

$$\|u\|_{L^\infty(0,t;L^{\vartheta+p+1})} \leq C \|u_0\|_{L^{\vartheta+p+1}} + C t^{\frac{p}{p+\vartheta+1}} E(0)^{\frac{1}{p+\vartheta+1}}, \quad \forall t > 0.$$

Since $p+\vartheta+1 \geq 2,$ by Hölder's inequality it holds

$$\begin{aligned} \|u\|_{L^\infty(0,t;L^2)} &\leq |\Omega|^{\frac{p+\vartheta-1}{2(p+\vartheta+1)}} \|u\|_{L^\infty(0,t;L^{\vartheta+p+1})} \\ &\leq C |\Omega|^{\frac{p+\vartheta-1}{2(p+\vartheta+1)}} \left(\|u_0\|_{L^{\vartheta+p+1}} + t^{\frac{p}{p+\vartheta+1}} E(0)^{\frac{1}{p+\vartheta+1}} \right), \forall t > 0. \end{aligned} \tag{19}$$

where $|\Omega|$ denotes the Lebesgue measure of $\Omega.$ From (16), it holds

$$\|u\|_{L^\infty(0,t;L^2)} \geq \|u_0\| + C\left(e^{ct} - 1\right), \quad \forall t > 0. \tag{20}$$

Both the facts (19) and (20) hold, provided that

$$\|u_0\| + C\left(e^{ct} - 1\right) \leq C |\Omega|^{\frac{p+\vartheta-1}{2(p+\vartheta+1)}} \left(\|u_0\|_{L^{\vartheta+p+1}} + t^{\frac{p}{p+\vartheta+1}} E(0)^{\frac{1}{p+\vartheta+1}} \right), \quad \forall t > 0,$$

which is a contradiction when t is sufficiently large. Thus, the local solutions blow up in a finite time. The proof is complete.

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