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Outliers and Structural Breaks Detection in Volatility Data: A Simulation Study using Step Indicator Saturation

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ABSTRACT

It is known that the presence of structural breaks and outliers in the data series will distort the analysis in many aspects such as estimation and the accuracy of the forecast. Therefore, the present study aims to detect outliers and structure breaks simultaneously in simulated volatility data via GARCH model using step indicator saturation (SIS). The procedure begins with the detection of outliers in simulated volatility data using various significant levels to determine the suitability on the different number of observations. The accuracy of the detections is accessed using multiple indicators such as potency, gauge, misclassification rate and false discovery proportion. The suitability of significant level is then recommended to be applied to the next procedure of the detection of structural break and outliers in the simulated volatility data.

Keywords: Outliers, Structural breaks, GARCH

INTRODUCTION

Volatility is one of the crucial elements of risk management. It is also being used widely as a basic measure of total risk in the financial asset where it plays a vital role in financial asset management. For example, trading in stocks by investors. Investors can raise capital, pay off debt as well as gaining profit from dividend distributed by involving in stock trading. However, this depends solely on the stock market trend. The stock market can be bullish (rising) in a trend or bearish (falling) in a trend. A market that experiences a bullish trend attracts more investors which indirectly give a positive impact on the direction of the economy. On the other hand, the opposite effect happened when the stock market is experiencing a fall. This uncertainty is referred to as the volatility of the financial market.

Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models are the most common model in assessing the volatility in financial time series data (Carapole and Zekokh, 2019). According to Guesmi et al. (2013), among various types of volatility models, GARCH model is yet found to be the best method in modelling the volatility. Despite the popularity of the GARCH model, it is often observed that the estimated residuals estimated from their mean equation still have excess kurtosis (Baillie and Bollerslev, 1989). Several attempts have been made to overcome the problem by allowing the error to follow a conditionally fat-tailed t-distribution (Bollerslev, 1987). However, despite using t-distribution to the error terms, the problem of excess kurtosis continues to exist.

The possible cause to the excess kurtosis is the presence of additive outliers that are not captured by a GARCH model (Balke and Fomby (1994) and Franses and Ghijsels (1999). The additive outliers refer to the situation in which a remarkably large or small value is occurring for a single observation (Fox, 1972). In general, it can be interpreted as measurement errors or as an impulse effect due to the unexpected events, such as a strike, accident or a breakdown and so forth without carry-over to other subsequent values of the time series (Ledolter (1989) and Pena (2000)). Carnero and Pena (2006) advised checking if the series containing outliers before fitting the GARCH model type.

Moreover, some of the researchers also argued that the classical GARCH model could not appropriately capture some type of persistence displays by the volatility of financial data. The reason for this occurrence is the failure to take into account the structural shift in the model (Diebold (1986) and Lamoureux and Lastrapes (1990). As simulated by Andreou and Ghysels (2002), strongly-persistent volatility can lead to essential size distortions in structural break tests. Neglecting such a break can generate spuriously measured persistence with the sum of the estimated autoregressive parameters of the conditional variance heavily biased towards one. Lamoureux and Lastrapes (1990) and Fang and Miller (2009) found that high volatility persistence measured by the GARCH model disappears by including a dummy variable for the structural break.

Consequently, it seems rather imperative that the modelling of financial time series using the GARCH model should take into account the presence of outliers and structural breaks. The two approaches that seem to be focal in the relevant strands of literature are to incorporate breaks in the mean and/or volatility dynamics and to identify and correct for the presence of outliers before fitting a particular model. Thus, the main objective of this paper is to introduce the step indicator saturation (SSI) as a method that can detect outliers and structural breaks when modelling using GARCH model being done. However, this paper will focus on simulated data from the GARCH model.

METHODOLOGY

This section begins with the discussion on the step indicator saturation (SIS). Then a brief discussion about the GARCH model will be presented. Finally, the simulation procedure will be shown.

Step Indicator Saturation

Hendry (1999) introduced the step indicator saturation (SIS) a general-to-specific approach for an unknown number of breaks, occurring at unknown times, with unknown durations and magnitudes. Step indicators are defined as the accumulation of impulse indicators up to each of the next observation. The saturation setting of T-1 step-shift indicator is included in the regression model. Step indicators take the form of $i_1 = (1, 0, 0, \dots, 0)$, $i_2 = (1, 1, 0, \dots, 0)$,…, $i_{T-1} = (1, 1, 1, \dots, 1, 0)$. The "T" step of $i_T = (1, 1, 1, \dots, 1, 1)$ is the intercept.

The study of SIS is first developed by Doornik (2012) to consider the step-indicator saturation to capture the breaks. Using Monte Carlo simulation study, he provides the evidence of the feasibility of SIS in detecting breaks on various aspects such as the accuracy of detection when location shifts occur and improving in accepting frequency compared to the impulse indicator saturation (IIS). Then he compares to Chow (1960) tests.

This paper utilized SIS with only constant, *c* as a regressor following the work of the Castle et al. (2012). Let $y_t = c + \varepsilon_t$ where ε_t is normally and independently distributed with zero mean and variance as σ^2. Equation 1 shows the augmented block of impulse indicators.

$$y_t = c + \sum_{k=1}^{\frac{T}{2}} \delta_{lk} S_t(k) + \varepsilon_t. \tag{1}$$

Using the split-half approach, equation 1 contains the first T/2 parameter to be analyzes. Any indicator, S_t with *t*-value less than the critical value α is deleted. In the second step, the remaining half of the step indicator, S_t are estimated and eliminated. The selected step indicator, S_t from the terminal model are then combined and re-estimated to give the final model. SIS procedure distinguishes structural breaks as a segment of step indicators, S_t with the same sign and magnitudes. Large outliers are detected with the step indicator, S_t with different sign.

GARCH Model

Generalized autoregressive conditionally heteroscedastic (GARCH) models were introduced by Bollerslev (1986) as an extension of autoregressive conditionally heteroscedastic (ARCH) models by Engle (1982). An extensive discussion on both models can be found in the monograph by Francq and Zakoian (2019) and Xekalaki and Degiannakis (2010). The fundamental concept of the GARCH model is the conditional variance which is the variance conditional on the past. This paper employed the GARCH (1,1) as a benchmark for the volatility data. The reasons for using GARCH (1,1) because of its superiority predictive ability as reported by Hansen and Lunde (2005). The GARCH (1,1) allows the conditional variance to be dependent on its lags and given as below:

$$r_t = \delta + \varepsilon_t, \quad (2)$$

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \quad (3)$$

where $\varepsilon_t = \eta_t \sigma_t$, δ is the conditional mean, ε_t is independent and identically distributed with $N(0,1)$, ε_{t-1}^2 denotes the ARCH term, σ_t^2 is the conditional variance, σ_{t-1}^2 is the GARCH parameter where $\omega > 0$, $\alpha_1 \geq 0$ and $\beta_1 \geq 0$. The process is stationary if $\alpha_1 + \beta_1 < 1$. The persistence of volatility is measured as the sum of α and β.

Monte Carlo Simulation

The simulation procedure begins with simulating data for GARCH (1,1) under Gaussian distributions. The simulation setting for this study are as follows;

1. Parameters $\alpha_1 = 0.1$, $\beta_1 = 0.8$ and $\omega = 1 - \alpha_1 - \beta_1$.
2. Sample sizes, $T = 500, 1000, 2000, 3000$ observations.
3. Significance levels, $\alpha = 0.05, 0.025, 0.01$.
4. Error distributions: Gaussian distribution
5. Multiple additive outliers (AO) of different magnitude (3, 5, 10 and 15 standard deviations) are placed randomly in simulated return series in both positive and negative magnitude.

The restriction $0 < \hat{\alpha}_1 + \hat{\beta}_1 \leq 1$ and $\hat{\alpha}_0 > 0$ are imposed in the estimation procedure. The parameter chosen is similar to the study by Carnero et al. (2012) and Marczak and Proietti (2016). In this study, the simulations are carried out using four different sample sizes; $T = 500$, 1000, 2000 and 3000 to reflect the standard sample sizes for financial time series data. The sample size of 3000 corresponds to more than ten years of daily data and usually considered enough to understand the variability of the data.

The simulation study intends to understand the suitable significance level to be used based on the number of sample size. The study carried out using three significance levels, $\alpha = 0.05, 0.025$ and 0.01. The smaller the value of significance level, the chance of retaining irrelevant indicator variables is relatively lower. The performance of the SIS approach in detecting outliers is assessed using the concept of gauge, potency, misclassification and false discovery rate using the confusion matrix as proposed by Marczak and Proietti (2016). The confusion matrix can be illustrated using the following table;

Table 1: Confusion matrix

Actual	**Decision**		**Total**
	No outlier	Outlier	
No outlier	A	B	T-n
Outlier	C	D	n
Total	A+C	B+D	T

A and D denote numbers of correct decisions in the cases of no outlier and one outlier (at a particular observation), respectively. B and C, on the other hand, summarize all false decisions when no outlier is present, and when there is an outlier (at a particular observation), respectively. The potency is then defined as the ratio D/n, which is known as the true positive rate (also called the hit rate, recall or sensitivity) in the classification literature. The gauge is given by the ratio B/[(T−n)], the so-called false positive rate (or false alarm rate). The misclassification rate is (B+C)/(n) and B/(B+D) is the false discovery proportion.

After finding the prefer significance level, the simulation continues in discovering structural breaks and outliers in the conditional variance. The design of the simulation study follows that of Hillebrand and Setala (2005) on the effects of neglecting parameter changes (α and β) in the estimation of the GARCH (1,1) model. This study only focuses on the correct detection of the timing of single structural breaks where one break happens in the middle of the simulated data. In terms of the sample size, this study will use the similar number with those reported in the previous outlier's simulation procedure (T= 500, 1000, 2000 and 3000) as it reflects the typical sample size in the financial time series analysis

For single breaks, the breaks are placed starting in the middle (0.50T)+1 of the sample for each sample size under study (T = 500, 1000, 2000 and 3000). In terms of parameter value for the GARCH (1,1) model, the parameter of the conditional mean equation is fixed at $\mu = 0$ and $\phi = 0.7$. From Table 2, the first half of the simulated data (0.50T) will follow the initial GARCH (1,1) model without breaks (Setting 1). In contrast, the second half of the simulated data (0.50T +1) will obey model with Setting 2, 3 or 4.

Table 2: The initial parameter values and parameter switches

Setting number	σ	ω	α	β	λ
1	1.00	0.20	0.10	0.70	0.80
2	1.73	0.26	0.15	0.70	0.85
3	2.83	0.34	0.18	0.70	0.88
4	4.20	0.42	0.20	0.70	0.90

Notes: ω is constant of the conditional variance for the GARCH (1,1) model. ω, α and β are chosen to generate the column of the table, holding other parameters fixed. λ is the sum of α and β.

RESULTS AND DISCUSSION

Figure 1 presents the simulated data for the different number of observations (T=500, 1000, 2000 and 3000) of GARCH (1,1) with the Gaussian error distribution. While Table 3 represents the outliers' location according to its size and magnitude. Then, the outliers are replaced at the random location, as stated in Table 3, where Figure 2 presents the data series contaminated by outliers.

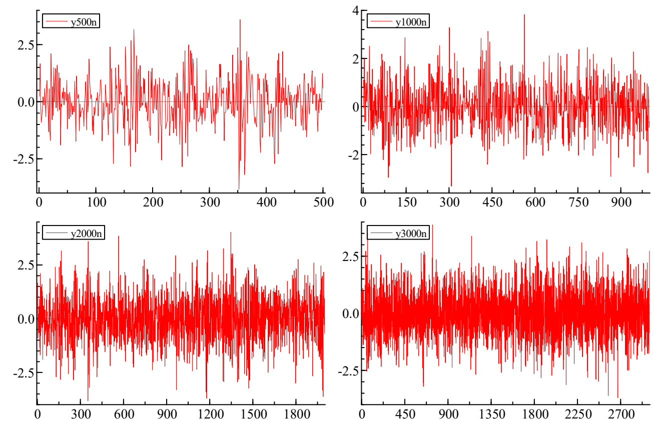

Figure 1: Simulated data series under Gaussian GARCH (1,1)

Table 3: Outliers and random location for Gaussian GARCH (1,1)

Number of observations	Outliers size	Outliers (value)	Random location (+ve)	Random location (-ve)
500	$\pm 3\sigma_y$	±3.0780	254	188
	$\pm 5\sigma_y$	±5.1300	441	232
	$\pm 10\sigma_y$	±10.2600	34	22
	$\pm 15\sigma_y$	±15.3900	259	500
1000	$\pm 3\sigma_y$	±2.9196	520	201
	$\pm 5\sigma_y$	±4.8655	307	316
	$\pm 10\sigma_y$	±10.3200	669	870
	$\pm 15\sigma_y$	±15.4800	104	50
2000	$\pm 3\sigma_y$	±3.1371	1661	868
	$\pm 5\sigma_y$	±5.2285	262	127
	$\pm 10\sigma_y$	±10.4570	1837	1862
	$\pm 15\sigma_y$	±15.6855	1457	885
3000	$\pm 3\sigma_y$	±2.9756	1960	1063
	$\pm 5\sigma_y$	±4.9594	364	1640
	$\pm 10\sigma_y$	±9.9188	2006	2897
	$\pm 15\sigma_y$	±14.8782	2382	1145

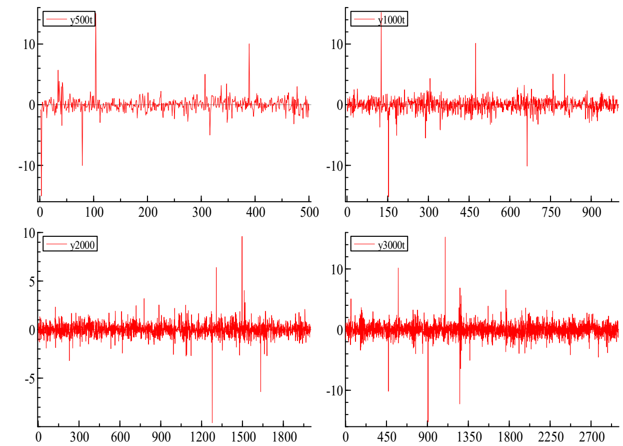

Figure 2: Contaminated simulation data series

Figure 3 to 6 summarize the simulation result on a various significance level for each performance indicator. The most suitable significance level is then recommended based on the sample size. The recommendation will provide a guideline for researchers in order to choose the correct significance level when using the SIS approach depending on their sample size. Figure 3 illustrates the summary of the potency rate for each number of simulated sample size. As the number of sample size increase, the potency rate for α = 0.05 and α = 0.025 increase substantially to 100%. The similar increasing pattern also recorded for α = 0.01 but reaching its maximum level at only 70% at the sample size of 3000.

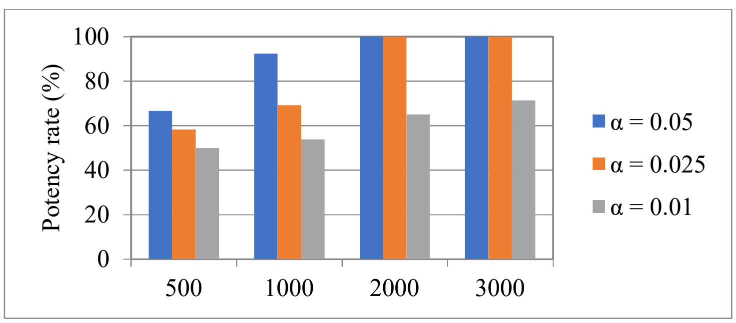

Figure 3: Summary of potency rate of GARCH (1,1)-Gaussian

Figure 4 summarizes the gauge rate for each significance level. It is interesting to note that the gauge rate is zero for the sample size of 500 and 1000. The gauge rate is zero at the smallest significance level, α = 0.01. It reflects the ability of SIS to identify the correct outliers with the minimum gauge rate. While Figure 5 presents the misclassification rate for each significance level on a different number of sample sizes. The result reveals that for the sample size of 500 and 1000, the smallest percentage of misclassification rates are recorded at α = 0.05. On the other hand, the minimum misclassification rate for the sample size of 2000 is recorded at α = 0.025. For the sample size of 3000, it is suggested that α = 0.01 should be applied to minimize the misclassification rate of outliers' detection.

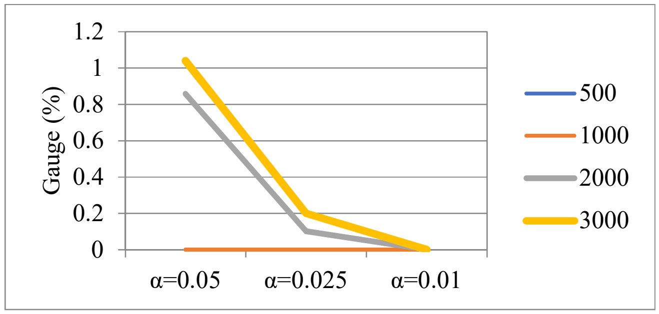

Figure 4: Summary of gauge of GARCH (1,1)-Gaussian

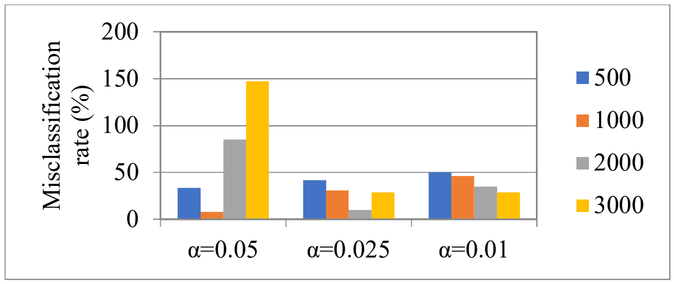

Figure 5: Summary of the misclassification rate of GARCH (1,1)-Gaussian

In the context of false discovery proportion, remarkable result with zero false discovery proportion is detected for the sample size of 500 and 1000. It is also interesting to discover that the remarkable performance of $\alpha = 0.01$ was discovered with zero false discovery proportion throughout all sample sizes. Figure 6 provides a summary of the false discovery proportion of GARCH (1,1)-Gaussian under different level of significance.

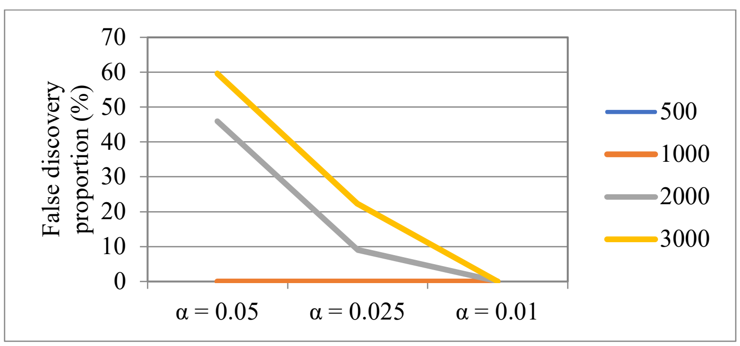

Figure 6: Summary of false discovery proportion of GARCH (1,1)-Gaussian

Based on four performance indicators, the study provides the recommended significance level based on the sample size. The recommendation will provide a guideline for researchers to choose the most suitable significance level when using the SSI on GARCH (1,1)-Gaussian depending on their sample size. The significance level, $\alpha = 0.05$ is recommended for the sample size of 500, $\alpha = 0.025$ for sample size of 2000 and $\alpha = 0.01$ for the sample size of 3000. From previous simulation finding, the study continues with a similar setting in term of a sample size to show the performance of SIS in detecting structural breaks in GARCH (1,1) simulated data. The setting is based on the discussion in the earlier section. Figure 7 displays the simulation data of the single break of GARCH (1,1) for the sample size of 500, 1000, 2000 and 3000 and the dashed line represent the unconditional variance change in the data series for three different models setting (refer to Table 2).

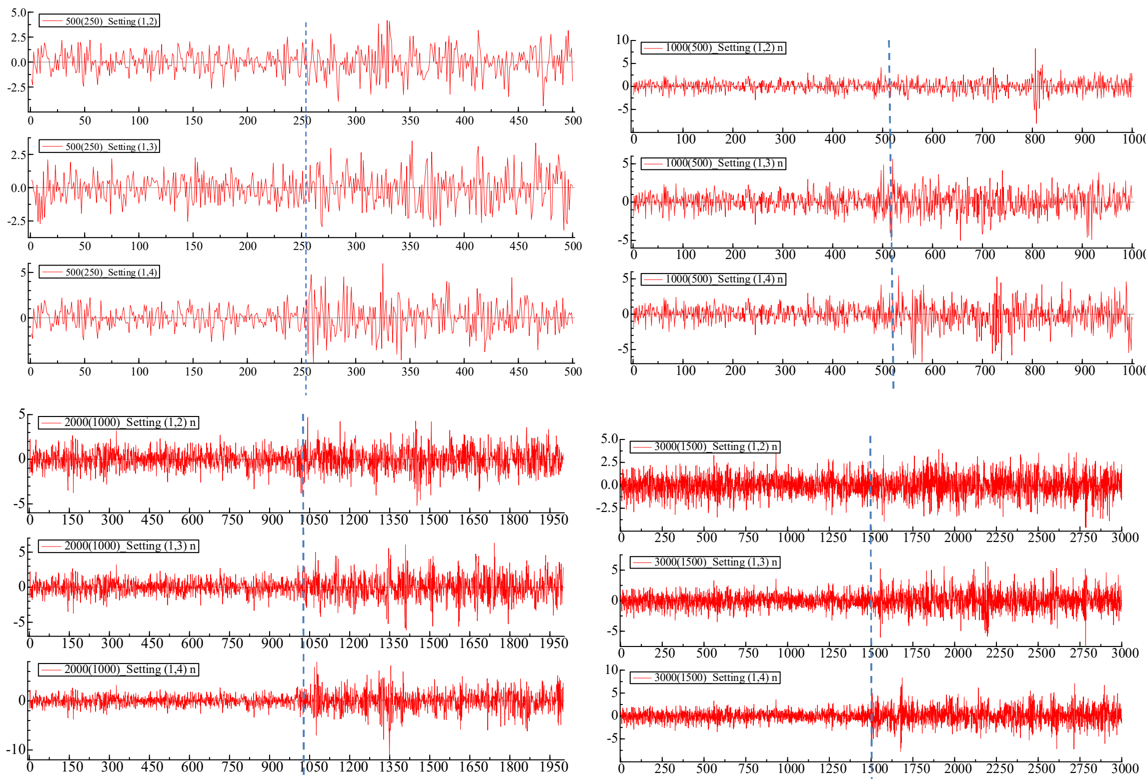

Figure 7: Single break of GARCH (1,1) under model setting (1,2), (1,3) (1,4)

Table 4: Results in detecting structural breaks and outliers

Sample size	Model setting	Num. of breaks	Location of breaks	Num. of outliers
500	(1,2)	1	283	15
	(1,3)	1	268	17
	(1,4)	1	258	20
1000	(1,2)	1	394	34
	(1,3)	1	483	27
	(1,4)	1	497	40
2000	(1,2)	1	1104	32
	(1,3)	1	1072	51
	(1,4)	1	1079	57
3000	(1,2)	1	1567	26
	(1,3)	1	1560	40
	(1,4)	1	1502	48

Table 4 tabulates the results of the SIS in detecting structural breaks and outliers for volatility data from GARCH (1,1). It is interesting to note that SIS detected one structural break consistent with our setting of one break in the simulated volatility data. Furthermore, as the sample size increases, the location of the breaks is near to the prespecifies location that is 251 for sample 500, 501 for sample 1000, 1001 for sample 2000 and 1501 for sample 3000. A similar pattern is also shown by model setting from setting (1,2) to (1,4). The number of outliers detected also increases as the sample size increases.

CONCLUSION

This study aims to present the performance of step indicator saturation (SIS) in detecting structural breaks and outliers in simulated volatility data from GARCH (1,1) model. The first stage of simulation study recommended a significant level to be used in detecting outliers in the data series. It will serve as a guideline for the researcher to pick the suitable significant according to the number of observations. While the second stage simulation study, show that SSI manages to detect single break as pre-assign for the simulated data. Moreover, the sample size and model specification affect the accuracy of the location detection of the structural break. Further research will focus on non-Gaussian GARCH model and using real data.

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