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Possibility Based TOPSIS with Inter Criteria Correlation and Similarity Measure

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ABSTRACT

The method of Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is a well-known decision making procedure and has been used to solve many real application problems. It involves a two stage process of determining the criteria weight and rating of alternatives using the concept of distance from the best and the worst ideal solutions. In the situation where many decision makers involved in the evaluation using TOPSIS, conflict of aggregation may happen when decision makers experience uncertainty due to incomplete or missing information and the correlation between criteria is ignored. In this paper, an improved TOPSIS method under fuzzy environment is proposed by combining consensus possibility measures to overcome the issue of conflict of aggregation and the Criteria Importance through Inter-Criteria Correlation (CRITIC) technique is integrated to include the correlation between the criteria in the computation. The CRITIC technique has the ability to contrast intensity and conflict among the criterion used in the evaluation. In obtaining the final output, the distance based similarity measure is incorporated to minimize the loss of information. The consistency of ranking of the proposed TOPSIS is compared and analyzed with other existing fuzzy TOPSIS methods using some variation of similarity measures.

Keywords: TOPSIS, CRITIC, Consensus Possibility Measure.

INTRODUCTION

Technique for Order Performance by Similarity Ideal Solution (TOPSIS) [1] was first introduced by Hwang and Yoon (1981) as one of the practical and useful techniques for ranking and selection of feasible option. It is based on the idea of the positive ideal solution (PIS) and negative ideal solution (NIS) in which the chosen alternative should have the shortest distance from PIS and the farthest from NIS. Simultaneously, TOPSIS also considers the distance to both PIS and NIS, and a preference order is ranked according to the relative closeness that taking into consideration the two distances. The higher the value of the closeness coefficient of an alternative gives a higher ranking or preference to the alternative. In a fuzzy environment, the use of exact value is impossible and inadequate to model the real life scenario. Thus, a more realistic approach based on a linguistic assessment is developed (Chen, 2000) where linguistic terms were used in the evaluation. Some variation and extension of fuzzy TOPSIS have been further introduced to cater specific condition and requirement of the decision making problems (Jahanshahloo, 2006; Wang & Elhag, 2006; Chen & Tsao, 2008).

When the evaluation involved multi experts, conflict of aggregation may happen. Thus consensus needs to achieve in order to get the maximum agreement from all the experts. Nowadays, there are many different ways of measuring consensus had been developed. Some of the approaches are based on possibility measure, fuzzy linguistic information, average related to distance and qualitative reasoning based on entropy method. Possibility measure is one of the algorithm that is appropriately measured the degree of consensus agreement in fuzzy environment. Possibility measure helps in dealing with the difficulty of conflict aggregation in the consensus process and is adapted from the possibility theory (Noor-E-Alam et al., 2011). It is a practical approach in describing the degree of uncertainty involving human judgement. Recently, there are a few studies dealing with the consensus possibility measure have been introduced (Noor-E-Alam et al., 2011; Igoulalene & Benyoucef, 2013; Igoulalene & Benyoucef, 2014).

One of the steps in TOPSIS procedure is identifying the importance of the criteria weights and can be computed using the weighting method. This approach is classified into three categories which are subjective approach, objective approach and integrated approach (Jahan, 2012). There are two group of methods concerned under the subjective approach which are direct weighting procedure and pairwise comparison while the objective method can be classified into mean weight, entropy, standard deviation method, preference selection index and the Criteria Importance through Inter-Criteria Correlation (CRITIC) (Diakoulaki et al., 1995). Recently, some authors had combined both objective and subjective approaches. One of the latest integrated methods is Correlation Coefficient and Standard Deviation (CCSD) (Wang and Luo, 2010). The CRITIC method has an advantage over others as the criteria weight and their correlation to each other may be obtained merely through the evaluation of the alternatives with respect to each criterion. The issue of determining the TOPSIS criteria weight has been tackled by several researchers in many different ways (Wang & Hsu, 2004; Wang et al. 2010; Deng et al., 2012).

The integration of similarity measure in fuzzy TOPSIS procedure has been a common approach recently to minimize the loss information in the evaluation process. Similarity measure (SM) is a concept used to measure how alike two objects are. Various new SM functions had been proposed such as by Wang & Lee (2009), Luukka (2011) and Collan & Luukka (2014). Each of the SM function has its owns strengths and weaknesses. Type of SM function used depends on the situation and background of the problem since different types of SM are suitable for different types of analysis. There are three common types of SM functions which are distance based, featured based and probabilistic SM (Johanyak & Kovacs, 2005) and the simplest method to compute the similarity of fuzzy sets is based on their distance. Some of the distance based similarity measures have been introduced by several authors such as Hsieh & Chen (1999), Chen & Chen (2001), Wei & Chen (2009), Hejazi et al. (2011) and Vicente et al. (2013).

This paper proposes a fuzzy TOPSIS method combining the consensus possibility measure, CRITIC technique and distance based similarity function. Consensus possibility measure by Noor-E-Alam et al. (2011) is employed in order to overcome the conflict aggregation among a group of decision makers. Meanwhile, CRITIC technique by Diakoulaki et al. (1995) is applied in determining the importance of criteria weights. Similar distance features for both TOPSIS and SM is the reason why SM is being integrated in the proposed method where distance based SM is implemented instead of using normal distance or vertex method. A new distance based SM by Vicente et al. (2013) that has elements of distance between the centres of gravity, geometric distance, and a new term based on the shared area between the fuzzy numbers is employed in this study. A comparison on the consistency in the ranking between the developed new hybrid TOPSIS approach and the existing hybrid TOPSIS approach by Igoulalene & Benyoucef (2014) is performed.

PRELIMENARIES

A fuzzy set A in a universe discourse X can be defined mathematically by a membership function $\mu_A(x)$ referring to the grade or degree to which any element x in A belongs to the fuzzy set A. In other words, each element x in A is mapped to the real number in the interval [0, 1] by membership function. A fuzzy set A in a universe discourse X is known as a convex fuzzy set, implying that for all x_1, x_2 in X, $\mu_A(\lambda x_1+(1-\lambda)x_2)\geq \min(\mu_A(x_1),\mu_A(x_2))$ where $\lambda\in[0,1]$. Otherwise, the set is a non-convex fuzzy set. The fuzzy set A is a normal fuzzy set when at least one $x\in A$ attains the maximum membership degree where $\mu_A(x)=1$. A fuzzy subset of the universe discourse X that satisfies both convex and normal is known as a fuzzy number.

A trapezoidal fuzzy number $A=(a_1, a_2, a_3, a_4)$ where a_1, a_2, a_3 and a_4, are real values can be defined by a membership function $\mu_A(x)$ as follows (Zadeh, 1965):

$$\mu_A(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, & a_1\leq x\leq a_2,\\ 1 & a_2\leq x\leq a_3,\\ \dfrac{a_4-x}{a_4-a_3} & a_3\leq x\leq a_4,\\ 0 & \text{otherwise}\end{cases}$$

A fuzzy number $A=(a_1, a_2, a_3, a_4; w_A)$ is called a generalized trapezoidal fuzzy number with a_1, a_2, a_3 and a_4 are real values and $0\leq w_A\leq 1$ where w_A denotes the degree of confidence with respect to the decision makers' opinions. In particular, if $w_A=1$, then the generalized fuzzy number A is a normalized trapezoidal fuzzy number. Linguistic variable is a concept introduced by Zadeh (1965) and is represented by a quintuple (v, $T(v)$, X, G, M) where v is the name of the value, X denotes the universe discourse, which is linked with the base variable x, $T(v)$ is referred to the term set of v, which is the set of the name of linguistic value of v, G denotes as a syntactic rule for generating the name V, of values v and lastly M represents a semantic rule for linked with each X that can be expressed as $\widetilde{M}(v)$, and it is a fuzzy subset of X. A simple example of a linguistic variable is "Age" with linguistic terms such as "Very Young", "Young", "Middle Age", "Old" and "Very Old" with possible universe of discourse of the interval [0, 100].

In order to deal with the generalized trapezoidal fuzzy number, arithmetic operations are formulated based on the concept of function principle or the extension principle [27]. Let A and B be two generalized fuzzy numbers a such that $A=(a_1, a_2, a_3, a_4; w_A)$ and $B=(b_1, b_2. b_3, b_4; w_B)$, then, the arithmetic operations are described as:

- Addition: $A+B=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4;\min(w_A,w_B))$
- Subtraction: $A-B=(a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1;\min(w_A,w_B))$
- Multiplication: $A\times B=(a_1\times b_1,a_2\times b_2,a_3\times b_3,a_4\times b_4;\min(w_A,w_B))$
- Division: $A/B=(a_1/b_4,a_2/b_3,a_3/b_2,a_4/b_1;\min(w_A,w_B))$

For the addition and subtraction operations, the values of a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4 are real values. On the other hand, a_1, a_2, a_3, a_4, b_1, b_2. b_3 and b_4 are all positive real numbers for multiplication while for division a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4 are non-zero positive numbers.

CONSENSUS POSSIBILITY MEASURE

In this study, possibility measure from Igoulalene & Benyoucef (2013) is utilized as an algorithm performed for conflict aggregation that integrates the possibility theory of fuzzy logic with a maximal containment method. From the concept of possibility theory of fuzzy logic, the relationship of the probability quantifier (P_S), possibility quantifier (Po_S) and constant (w_s) with $w_s \leq 1$ for each criterion can be addressed as

$$\sum_{\forall S} P_S \times Po_S \leq w_S \tag{1}$$

and the constant equation is represented as

$$w_S = Min_S\left\{1 - P_S + P_S^2\right\}. \tag{2}$$

In order to convert probability into possibility, the relationship between probability and possibility can be explained as in equation (3) where U is the possibility transfer bound.

$$Po_S = P_S + U \tag{3}$$

where U can be derived from the equation (1) and (3) as

$$U \leq \frac{w_S - \sum_S (P_S)^2}{\sum_S (P_S)} \tag{4}$$

The possibility quantifier that is transferred from probability quantifier is then treated as a truth-value (TV). By applying the maximal containment method, the TV is transformed into linguistic truth-value (LTV). The Certainty Compliance (CC) is obtained for aggregation purposes by incorporating the optimism index represented by $I_r = \gamma B_r^L + (1-\gamma) B_r^U$ and the value of γ can be any value between the interval [0, 1]. This CC function can be described as

$$CC = \frac{\sum_{i=1}^{n} \ln(I_r)}{n}$$

where

$$\ln(I_r) = \max\left(\min\left(\frac{I_r - 0}{0.5 - 0}, \frac{1 - I_r}{1 - 0.5}\right), 0\right)$$

is the information content of the value I_r.

CRITERIA IMPORTANCE THROUGH INTER-CRITERIA CORRELATION (CRITIC)

In order to determine the importance weight of different criteria j denoted by w_j, a weighting method of Criteria Importance through Inter-Criteria Correlation (CRITIC) (Diakoulaki et al., 1995) is applied. This technique is not only considering the standard deviation of every criterion, but the correlations among the criteria is also being accounted. The dependency between two variables can be measured using correlation and the steps for obtaining the importance of criteria are given as follows. Let $\tilde{x}_{ij}$ be the evaluation of the importance of criteria j by decision maker i. Then

Step 1: Normalize the criteria using

$$\begin{cases} r_{ij} = \dfrac{\tilde{x}_{ij} - \tilde{x}_j^{\min}}{\tilde{x}_j^{\max} - \tilde{x}_j^{\min}}, & i = 1,\ldots,m;\ j = 1,\ldots n, \quad r_{ij} \in B \\ r_{ij} = \dfrac{\tilde{x}_j^{\max} - \tilde{x}_{ij}}{\tilde{x}_j^{\max} - \tilde{x}_j^{\min}}, & i = 1,\ldots,m;\ j = 1,\ldots n, \quad r_{ij} \in C \end{cases} \tag{5}$$

where B and C are sets of benefit and cost criteria respectively.

Step 2: Calculate the correlation ρ_{jk} between criteria as

$$\rho_{jk} = \frac{\sum_{i=1}^{m}\left(r_{ij} - \bar{r}_j\right)\left(r_{ik} - \bar{r}_k\right)}{\sqrt{\sum_{i=1}^{m}\left(r_{ij} - \bar{r}_j\right)^2 \sum_{i=1}^{m}\left(r_{ik} - \bar{r}_k\right)^2}} \qquad j,k = 1,\ldots,n \tag{6}$$

Step 3: Determine the amount of information c_j of the correlation using

$$c_j = \sigma_j \sum_{k=1}^{n}(1 - \rho_{jk}) \qquad j = 1,\ldots,n \tag{7}$$

Step 4: Compute the weight of each criteria j as

$$w_j = \frac{c_j}{\sum_{k=1}^{n} c_k} \qquad j = 1,\ldots,n \tag{8}$$

SIMILARITY MEASURE

Let two generalized trapezoidal fuzzy numbers A and B be represented by $A = (a_1, a_2, a_3, a_4; w_A)$ and $B = (b_1, b_2, b_3, b_4; w_B)$ where w_A and w_B are the height of the fuzzy numbers A and B respectively. The degree of similarity between A and B (Vicente et al., 2013), denoted by $SM(A,B)$ can be described as follows:

If $\max\{(a_4 - a_1), (b_4 - b_1)\} \neq 0$,

$$SM(A,B)=\left(1-\left|w_A-w_B\right|\right)\times\left(1-(1-\beta-\delta)\times\left(1-\frac{\int_0^1\mu_{A\cap B}(x)dx}{\int_0^1\mu_{A\cup B}(x)dx}\right)-\beta\frac{\sum\left|a_i-b_i\right|}{4}-\delta l_\infty\left[\left(X_A^*,Y_A^*\right),\left(X_B^*,Y_B^*\right)\right]\right) \quad (8)$$

Otherwise;

$$SM(A,B)=\left(1-\left|w_A-w_B\right|\right)\times\left(1-\left(\frac{1-\beta-\delta}{2}+\beta\right)\times\frac{\sum\left|a_i-b_i\right|}{4}-\left(\frac{1-\beta-\delta}{2}+\delta\right)\times l_\infty\left[\left(X_A^*,Y_A^*\right),\left(X_B^*,Y_B^*\right)\right]\right) \quad (9)$$

In both functions, the value of β and δ can be any positive value that satisfy the condition $\beta+\delta<1$. For simplicity, in this paper, $\beta=\delta=1/3$ is chosen. Other nomenclatures in (8) and (9) are given as follows:

- $\mu(x)$ is the membership function of x
- $l_\infty\left[\left(X_A^*,Y_A^*\right),\left(X_B^*,Y_B^*\right)\right]=\max\left\{\left|X_A^*-X_B^*\right|,\left|Y_A^*-Y_B^*\right|\right\}$
- $\mu_{A\cap B}(x)=\min_{0\le x\le 1}\left\{\mu_A(x),\mu_B(x)\right\}$
- $\mu_{A\cup B}(x)=\max_{0\le x\le 1}\left\{\mu_A(x),\mu_B(x)\right\}$
- Centroids of A and B, (x_A^*,y_A^*) and (x_B^*,y_B^*) is calculated as

$$y_A^*=\begin{cases}\dfrac{w_A\left(\dfrac{a_3-a_2}{a_4-a_1}+2\right)}{6} & \text{if } a_1\ne a_4 \text{ and } 0<w_A\le 1,\\ \dfrac{w_A}{2} & \text{if } a_1=a_4 \text{ and } 0<w_A\le 1,\end{cases}$$

$$x_A^*=\begin{cases}\dfrac{y_A^*(a_3+a_2)+(a_4+a_1)((w_A-y_A^*)}{2w_A} & \text{if } w_A\ne 0\\ \dfrac{a_4+a_1}{2} & \text{if } w_B=0\end{cases}$$

PROPOSED DECISION-MAKING PROCEDURE

A hybrid method of fuzzy TOPSIS, consensus-based possibility measure, CRITIC technique and distance based similarity measure for solving the decision making problems is proposed. The procedural steps of hybrid TOPSIS can be described as follows:

Step 1: Let $C = \{C_j \mid i = 1,..., n\}$ be a set of criteria and $A = \{A_i \mid i = 1,...,m\}$ be a set of alternatives under consideration. The evaluation of the criteria is carried out by a set of decision makers or experts $D = \{D_k \mid i = 1,..., l\}$. Here, the consensus among the experts is obtained as follows (Diakoulaki et al., 1995):

- Decide on the quantifier's set Q_s and each expert is invited to give their preferences with respect to each criterion and alternatives.
- Enumerate the probability quantifier P_s for different criteria.
- Determine the constant, w_s for every P_s and possibility transfer bound U using equation (2) and (4)..
- Every decision maker needs to select his or her $D_q \in [0,U]$. Here, D_q is assumed relative to the expert confident level, (E_{CL}), in answering the questionnaire as

$$D_q = \frac{E_{CL}}{10} \times U \tag{10}$$

- Subsequently, the possibility is computed using equation (3).
- Apply the Optimism Index, I_r to obtain crisp preferences.
- Compute the Certainty Compliance, CC for each decision maker to aggregate the criteria.
- Pick the experts' opinion that has the smallest value of CC for different criterion and alternative.

Step 2: Determine the importance of weights w_j of different criteria, C_j where $0 < w_j < 1$, $j = 1,2,..., m$ and $\sum_{j=1}^{m} w_j = 1$ using CRITIC technique which can be employed from the equation (5-8).

Step 3: Determine the linguistic scales of each fuzzy collective preference obtained from the consensus phase. Then, build the fuzzy decision matrix $\tilde{D} = [\tilde{x}_{ij}]_{m\times n}$ where $\tilde{x}_{ij}$ is rating of alternative A_i with respect to the criteria C_j.

Step 4: Transform the fuzzy collective decision matrix $\tilde{D}$ into weighted fuzzy collective preference matrix, $\tilde{V}$ by multiplying the importance weight of the evaluation criteria w_j and the values in the $\tilde{D}$.

Step 5: Identify the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS) denoted as

$$A_j^+ = \max_i (\tilde{v}_{ij}) \qquad A_j^- = \min_i (\tilde{v}_{ij}),$$

where A_j^+ refers to FPIS for each criterion C_j while A_j^- is FNIS for each criterion C_j.

Step 6: Calculate the fuzzy similarity measure of each alternative from the both ideal solutions for each criterion using

$$SM_{ij}^{+} = SM(\tilde{v}_{ij}, \tilde{v}_j^{+}) \qquad SM_{ij}^{-} = SM(\tilde{v}_{ij}, \tilde{v}_j^{-}), i = 1,\ldots,m,\; j = 1,\ldots,n$$

Based on the above expression, SM_{ij}^{+} is the similarity values between FPIS and weighted fuzzy preference while SM_{ij}^{-} is the similarity values between FNIS and weighted fuzzy preference for every criterion and alternative. The similarity measure of each alternative is computed using equation (8) and (9) and subsequently, the average of each alternative is calculated as

$$SM_i^{+} = \frac{\sum_{j=1}^{n} SM_{ij}^{+}(\tilde{v}_{ij}, \tilde{v}_j^{+})}{n}, \quad SM_i^{-} = \frac{\sum_{j=1}^{n} SM_{ij}^{-}(\tilde{v}_{ij}, \tilde{v}_j^{-})}{n} \text{ for all } i = 1,\ldots,m.$$

Step 7: Compute the similarity based closeness coefficient, $CCSM_i$ of alternatives. This is calculated as:

$$CCSM_i = \frac{SM_i^{+}}{SM_i^{+} + SM_i^{-}}.$$

Step 8: Rank all the alternatives in descending order, according to the similarity based closeness coefficient. The best alternative has the highest value of $CCSM_i$.

ILLUSTRATIVE EXAMPLE

In order to compare the ranking consistency of the new hybrid TOPSIS method that integrated consensus possibility measure, CRITIC technique and distance based similarity measure with an existing TOPSIS, an illustrative example by Diakoulaki et al., (1995) is being used as a benchmark. This example is based on the plant selection problem. In selecting the best alternatives which are three potential locations A_1, A_2 and A_3, four expert opinions correspondingly E_1, E_2, E_3 and E_4 are taken. Six criteria are observed which are skilled workers (C_1), expansion possibility (C_2), availability of acquirement material (C_3), investment cost (C_4), transport facilities (C_5) and climate (C_6). All the criteria are benefiting attributes except for criterion C_4 is the cost attribute.

The quantifiers and trapezoidal fuzzy numbers for experts' preferences and information processing, quantifier's sets for each criterion and experts' preferences given in Table 1, Table 2 and Table 3 are implemented to the algorithm of developed procedure. Thus, after applying all the procedural steps in the developed hybrid TOPSIS, the consistency of ranking between developed hybrid TOPSIS and existing hybrid TOPSIS can be compared.

Table 1: Number for Experts' Preferences Quantifiers and Information Processing Quantifiers

Expert Preferences Quantifiers	Fuzzy Numbers	Information Processing Quantifiers	Fuzzy Numbers
Very Poor (VP)	(0,0,0.1,0.2)	Absolutely False (AF)	(0,0,0.05,0.1)
Medium Poor (MP)	(0.1,0.2,0.2,0.3)	Mostly False (MF)	(0,0.1,0.1,0.2)
Medium Fair (MF)	(0.2,0.3,0.4,0.5)	Quite False (QF)	(0.1,0.15,0.25,0.3)
Fair (F)	(0.4,0.5,0.5,0.6)	Probably False (PF)	(0.2,0.3,0.3,0.4)
Medium Good (MG)	(0.5,0.6,0.7,0.8)	Somewhat False (SF)	(0.3,0.35,0.45,0.5)
Good (G)	(0.7,0.8,0.8,0.9)	Not Sure (NS)	(0.4,0.5,0.5,0.6)
Very Good (VG)	(0.8,0.9,1,1)	Somewhat True (ST)	(0.5,0.55,0.65,0.7)
		Probably True (PT)	(0.6,0.7,0.7,0.8)
		Quite True (QT)	(0.7,0.75,0.85,0.9)
		Mostly True (MT)	(0.8,0.9,0.9,1)
		Absolutely True (AT)	(0.9,0.95,1,1)

Table 2: Quantifier's Sets

Criteria, C_j	Quantifier's Set, Q_s
C_1	(VP,MF,F,G)
C_2	(MF,F,MG,G)
C_3	(F, MG,G)
C_4	(MF,F,MG,G)
C_5	(VP,MF,F,MG,G)
C_6	(MP,MF,F,MG,G)

Table 3: Experts' Preferences of Alternatives with Respect to each Criterion

A_i	C_j	E_1	E_2	E_3	E_4
A_1	C_1	MF	MF	F	F
	C_2	MG	G	MG	MG
	C_3	F	F	F	MG
	C_4	MF	F	F	F
	C_5	MF	F	F	MF
	C_6	F	F	MF	MF
A_2	C_1	G	G	G	F
	C_2	F	F	MG	MG
	C_3	MG	MG	MG	G
	C_4	MG	MG	G	G
	C_5	G	G	G	MG
	C_6	MG	G	MG	G
A_3	C_1	F	F	F	F
	C_2	MG	MG	MG	MG
	C_3	MG	MG	MG	MG
	C_4	F	F	F	F
	C_5	MG	MG	MG	MG
	C_6	F	F	MF	F

Finally, the comparison of similarity closeness coefficient and ranking for all alternatives using some existing similarity functions and the proposed one is presented in Table 4.

Table 4: Comparison of Similarity Closeness Coefficient and Ranking for all Alternatives using all Similarity Functions

	Similarity Functions									
	Proposed		Hsieh & Chen (1999)		Chen & Chen (2001)		Wei & Chen (2009)		Hejazi et al. (2011)	
A_i	$CCSM_i$	*Rank*	$CCSM_i$	*Rank*	$CCSM_i$	*Rank*	$CCSM_i$	*Rank*	$CCSM_i$	*Rank*
A_1	0.42572	3	0.49227	3	0.47166	2	0.49174	3	0.49152	3
A_2	0.57431	1	0.50773	1	0.50230	1	0.50826	1	0.50848	1
A_3	0.46239	2	0.49402	2	0.46650	3	0.49311	2	0.49201	2

It is agreeable that the most suitable alternative among the three potential locations is A_2 since alternative A_2 gives the highest value of closeness coefficient. Meanwhile, the overall performance ranking by this similarity function is $A_2 \succ A_3 \succ A_1$. Besides, other similarity functions (Hsieh & Chen, 1999; Chen & Chen, 2001; Wei & Chen, 2009, Hejazi et al., 2011) give the same results in term of the preferred alternative and ranking order. However, the similarity function given by Chen & Chen (2001) has a slight different ranking order due to different elements in comparing used in the similarity evaluation. The proposed similarity measure outperforms other similarity functions since a sufficiently comprehensive elements of the shared area, geometric distance and distance from the centre of gravity are included, hence most of the relevant information can be preserved. This can overcome the shortcomings of others similarity measures such that the parameters used are not always best suited to the certain cases or situation and the type of the fuzzy number that the model used.

In addition, by comparing this hybrid TOPSIS with the existing TOPSIS method, the results obtained for the preferred alternative and the overall ranking are similar. Thus, it can be verified that the ranking of developed hybrid TOPSIS is consistent with the existing hybrid TOPSIS although the ranking order for novel hybrid TOPSIS is determined by distance based similarity closeness coefficient whereas for existing hybrid TOPSIS is according to the distance closeness coefficient. The comparison of ranking and closeness coefficient is given in the Table 5.

Table 5: Comparison of Ranking Between Closeness Coefficient with Distance Based and Closeness Coefficient with Similarity Measure Distance Based

	Closeness Coefficient with Similarity Measure Based TOPSIS		Closeness Coefficent with Distance Based TOPSIS	
A_i	$CCSM_i$	Rank	CCD_i	Rank
A_1	0.42572	3	0.09	3
A_2	0.57431	1	0.12	1
A_3	0.46239	2	0.1	2

The similarity measures used in the proposed procedure consider the component of the perimeter, area, shared area, geometric distance and distance of centre of gravity of the fuzzy numbers.

Most properties incorporated in a fuzzy number are included in the similarity and hence it can minimize the loss of relevant information as compared to the distance based fuzzy TOPSIS where the defuzzification of fuzzy numbers is required to compute the distance between the resulting crisp numbers. This simplification may cause a severe omission of information in the evaluation (Niyigena et al., 2012).

CONCLUDING REMARKS

A hybrid TOPSIS that combines consensus possibility measure, CRITIC technique and distance based similarity measure is proposed. The method considers decision makers risk preference by employing the possibility measure. Apart from that, the distance based similarity measure is used to prevent loss of relevant information incorporated into the size and shape of the fuzzy numbers. It is observed that the proposed procedure is proven to be consistent in terms of overall ranking with the existing approaches. Meanwhile, the result of ranking in the implementation of the novel hybrid TOPSIS in a selection problem is seen to be relevant to the real life scenario. Thus, this method can be used as one of the decision making tools for MCDM problems in ranking and selecting the most suitable alternative. Different types of similarity measure may be used as a variant to the proposed method and also a different type of technique may be used in evaluating the correlation or dependencies between the criteria.

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REFERENCES

Bojadziev, G. and Bojadziev M. (2007). *Fuzzy logic for business, finance, and management*. Vol. **23**. World Scientific, 2007.

Chen, C. T. (2000). Extensions of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy Sets and Syst.*, **114(1)**: 1-9.

Chen, S. H. (1985). Fuzzy linear combination of fuzzy linear function principle. *Tamkang Jour. of Man. Sc.*, **6(1)**: 13-26.

Chen, S. J. & Chen, S. M. (2003). Fuzzy risk analysis based on similarity measures of generalized fuzzy numbers. *IEEE Trans. on Fuzzy Syst.*, **11(1)**: 45-56.

Chen, T. Y. & Tsao, C. Y. (2008). The interval-valued fuzzy TOPSIS method and experimental analysis. *Fuzzy Sets and Syst.*, **159(11)**: 1410-1428.

Cheng, C. H., Yang, K. L. & Hwang, C. L. (1999). Evaluating attack helicopters by AHP based on linguistic variable weight. *Eur. Jour. of Oper. Res.*, **116(2)**: 423-435.

Collan, M. & Luukka, P. (2013). Evaluating R&D projects as investments by using an overall ranking from four new fuzzy similarity measure-based TOPSIS variants. *IEEE Trans. on Fuzzy Syst.*, **22(3)**: 505-515.

Diakoulaki, D., Mavrotas, G. & Papayannakis, L. (1995). Determining objective weights in multiple criteria problems: The CRITIC method. *Comp. & Oper. Res.*, **22(7)**: 763-770.

Hejazi, S. R., Doostparast, A. & Hosseini, S. M. (2011). An improved fuzzy risk analysis based on a new similarity measures of generalized fuzzy numbers. *Exp. Syst. with App.*, **38(8)**: 9179-9185.

Hsieh, C. H. & Chen, S. H. (1999). A model and algorithm of fuzzy product positioning. *Info. Sc.*, **121(1-2)**: 61-82.

Hwang, C. L., & Yoon, K. (1981). Methods for multiple attribute decision making. In *Multiple Attribute Decision Making* (pp. 58-191). Springer, Berlin, Heidelberg.

Igoulalene, I., Benyoucef, L. & Tiwari, M. K. (2015). Novel fuzzy hybrid multi-criteria group decision making approaches for the strategic supplier selection problem. *Exp. Syst. with App.*, **42(7)**: 3342-3356.

Jahan, A., Mustapha, F., Sapuan, S. M., Ismail, M. Y. & Bahraminasab, M. (2012). A framework for weighting of criteria in ranking stage of material selection process. *The Int. Jour. of Adv. Man. Tech.*, **58(1-4)**: 411-420.

Jahanshahloo, G. R., Lotfi, F. H. & Izadikhah, M. (2006). Extension of the TOPSIS method for decision-making problems with fuzzy data. *App. Math. and Comp.*, **181(2)**: 1544-1551.

Johanyák, Z. C. & Kovács, S. (2005). Distance based similarity measures of fuzzy sets. *Proceedings of SAMI*, *2005*.

Luukka, P. (2011). Fuzzy similarity in multi-criteria decision-making problem applied to supplier evaluation and selection in supply chain management. *Adv. in Art. Intel.*, *2011*.

Niyigena, L., Luukka, P. & Collan, M. (2012). Supplier evaluation with fuzzy similarity based fuzzy TOPSIS with new fuzzy similarity measure. In *2012 IEEE 13th International Symposium on Computational Intelligence and Informatics (CINTI)* (pp. 237-244). IEEE.

Noor-E-Alam, M., Lipi, T. F., Hasin, M. A. A. & Ullah, A. S. (2011). Algorithms for fuzzy multi expert multi criteria decision making (ME-MCDM). *Knowledge-Based Syst.*, **24(3)**: 367-377.

Vicente, E., Mateos, A. & Jiménez, A. (2013). A new similarity function for generalized trapezoidal fuzzy numbers. In *International Conference on Artificial Intelligence and Soft Computing* (pp. 400-411). Springer, Berlin, Heidelberg.

Wang, T. C. & Lee, H. D. (2009). Developing a fuzzy TOPSIS approach based on subjective weights and objective weights. *Exp. Syst. with App.*, **36(5)**: 8980-8985.

Wang, T. C., Chang, J. F., Anh, T. N. & Chang, W. T. (2010, September). Applying TOPSIS method to evaluate the business operation performance of Vietnam listing securities companies. In *2010 International Conference on Computational Aspects of Social Networks* (pp. 273-277). IEEE.

Wang, Y. M. & Elhag, T. M. (2006). Fuzzy TOPSIS method based on alpha level sets with an application to bridge risk assessment. *Exp. Syst. with App.*, **31(2)**: 309-319.

Wang, Y. M. & Luo, Y. (2010). Integration of correlations with standard deviations for determining attribute weights in multiple attribute decision making. *Math. and Comp. Modelling*, **51(1-2)**: 1-12.

Wei, S. H. & Chen, S. M. (2009). A new approach for fuzzy risk analysis based on similarity measures of generalized fuzzy numbers. *Exp. Syst. with App.*, **36(1)**: 589-598.

Zadeh, L. A. (1965). Fuzzy sets. *Inf. and Control*, **8(3)**: 338-353.