

Difference Bound Matrix: A Different View of Fuzzy Autocatalytic Set

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ABSTRACT

Fuzzy Autocatalytic Set is a concept emerged from integration of graph theory, fuzzy theory and Autocatalytic Set theory. The concept has been used in modelling complex system such as clinical waste incineration process where the model is represented in a form of weighted directed graph. On the other hand, potential graph is a weighted directed graph represented by Difference Bound Matrix (DBM) which is used to represent timed automata in real time system. Since there is a similarity between potential graph and FACS, further investigation on the other matrix representation of FACS is made to expand the existing matrix representation of FACS by looking into the DBM. This paper provides a new definition on the DBM of FACS which leads to the formation of several theorems and corollaries. The representation of FACS by using DBM in modelling the clinical waste incineration system is also presented.

Keywords: Timed Automata, Autocatalytic Set, Pressurized Water Reactor, Directed Graph

INTRODUCTION

Fuzzy Autocatalytic Set (FACS) is an integration concept which combined three different theories namely fuzzy theory, autocatalytic set theory and graph theory (Sabariah, 2005). Fuzzy theory is a well-known to be good in explaining situation that relates to ambiguous, subjective and imprecise judgment (Daud et al., 2012) whilst autocatalytic set is an established concept used firstly to describe a catalytic process of chemical reaction (Noor Aini and Sumarni, 2017). Graph theory is a branch of mathematics that deals with many problems related not only to network (Zuraida et al. 2012), (Sabariah, 2005) but MCDM as well (Daud et al., 2005). Combination of those well-established theories which later known as FACS is represented as a directed graph and has been applied to modelled clinical waste incineration process (Sabariah et al., 2009) combustion process of a circulating fluidized bed boiler (Sumarni, 2013), (Razidah et al., 2014) and pressurized water reactor (Azmirul et al., 2015). The models have been analysed by using different type of matrices namely adjacency, transition and Laplacian matrix (Tahir et al., 2010). Although various of matrices are successfully used in giving better explanation of the system, however, further investigation on the other matrix representation need to be explored in order to enriched matrix representation of the concept.

This paper proposed a different view of the concept by looking at Difference Bound Matrix (DBM) (Miné, 2001). DBM has been used in the study on abstract domain for static analysis and real time system but has not been explored in describing any dynamics complex system. Here, since there is a similarity between the graph of FACS and potential graph which represented by the DBM, thus a new matrix representation of FACS by using DBM is proposed. A brief literature review on FACS and DBM is given in the following Sections.

FUZZY AUTOCATALYTIC SETS

Fuzzy Autocatalytic Set is defined as a subgraph where each vertex of the graph has at least one entered link with membership value $\mu(e_{i,j}) \in (0,1], \forall e_i \in E$. Incineration process of a waste generated by health-care activities in Malacca (Sabariah, 2005) is modelled using this concept. The idea of using fuzzy graph of type-3 is included whereby both the vertex and edge sets are crisp, but the edges have fuzzy heads and tails. As for the incineration process, six chemical compounds that play important roles in the process are Waste (v_1), fuel (v_2), Oxygen (v_3), Carbon Dioxide (v_4), Carbon Monoxide (v_5), and Other gases including water (v_6). are represented as vertices and a directed link from vertex i to vertex j indicates that the chemical compound i catalyzes the production of chemical compound j . Description of the fuzzy head, fuzzy tail and fuzzy edges connectivity of the edges in the graph are explained in (Sabariah, 2005).

$$M_F = \begin{bmatrix} 0 & 0 & 0.06529 & 0 & 0 & 0.13401 \\ 0.00001 & 0 & 0 & 0 & 0 & 0 \\ 0.15615 & 0 & 0 & 0 & 0 & 0 \\ 0.51632 & 0.68004 & 0.63563 & 0 & 0.99999 & 0 \\ 0.00001 & 0.00001 & 0.00002 & 0 & 0 & 0 \\ 0.32752 & 0.31995 & 0.29906 & 0.00001 & 0 & 0 \end{bmatrix}$$

The fuzzy values of the edge's connectivity are then constituting the element of the above adjacency matrix of FACS and Figure 1 represent the corresponding FACS model for the clinical waste incineration process. The bigger the value of the edge connectivity between two nodes, the stronger the connection between the nodes.

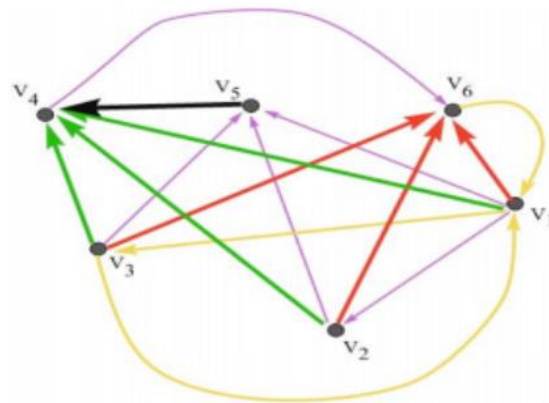


Figure 1. FACS model for the clinical waste incineration process

Besides, different colours of each edge indicate distinct membership value for the connectivity. The greater the membership value of the edge between the vertices, the heavier the link between them which implies the greater the connectivity between the vertices.

A DIFFERENCE BOUND MATRIX

Dill (1990) has introduced the Difference Bound Matrix (DBM) as constraint representation for confirmation of timed systems and is strongly used in checking the model of timed automata and timed Petri nets (Measche and Bethomieu, 1983). Each of the elements in the matrix represents a bound on the difference between two clocks. Furthermore, the matrix is only focused on abstract interpretation that can represent the new numerical abstract domain for static analysis and a finite representation of timed automata in real time system (Miné, 2001). It also used to represent system involving potential constraint (Péron and Halbwachs, 2007) The following definitions related to DBM are given as follow:

Definition 2.1: Difference Bound Matrix (Miné, 2001)

Difference Bound Matrix (DBM), M associated to the potential constraint set C is defined and presented as below:

$$M_{ij} = \begin{cases} c, & (v_j - v_i \leq c) \in C \\ +\infty, & \text{elsewhere} \end{cases}$$

Definition 2.2: Potential constraints (Miné, 2001)

Let $V = \{v_1, v_2, \dots, v_n\}$ be a finite set of variables with value in a numerical set \mathbb{I} and let $v_1 = 0$. Therefore, potential constraint can be written as $v_j - v_i \leq c$ where $v_j, v_i \in V$ and $c \in \mathbb{I}$. Furthermore, if $v_i \leq c$ and $v_i \geq c$, then it can be rewritten as $v_i - v_1 \leq c$ and $v_j - v_i \leq -c$ respectively.

Definition 2.3: Potential Graph (Miné, 2001)

A DBM, M is an adjacency matrix of directed graph $G(m) = G(V, A, w)$ with the edges weighted in \mathbb{I} where V is a set of nodes, $A \in V^2$ is a set of edge and $w: A \mapsto \mathbb{I}$ is weight function. Then, G is defined as:

$$G = \begin{cases} (v_i, v_j) \notin A, & \text{if } m_{ij} = +\infty \\ (v_i, v_j) \in A \text{ and } w(v_i, v_j) = m_{ij}, & \text{if } m_{ij} \neq +\infty \end{cases}$$

Example of a DBM and its corresponding potential graph of a set of potential constraints is given below where (a) A set of constraint, (b) the DBM and (c) a related potential graph:

The illustration of the concept is as below.

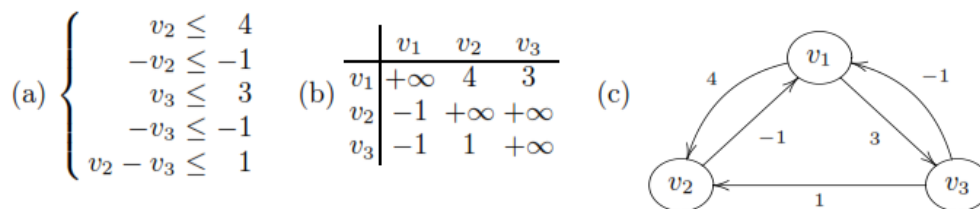


Figure 2. A DBM and its corresponding potential graph

DIFFERENCE BOUND MATRIX OF FACS

By using Definition 2.2, the definition of potential constraint of FACS is defined as follows:

Definition 3.1: Potential constraints of FACS

Let $G_{FT3}(V, E)$ be a no loop of FACS where $V = \{v_1, v_2, \dots, v_n\}$ be a set of vertices and $E = \{e_{ij}; i, j = 1, 2, \dots, n\}$ be a set of edges. A set of potential constraints C for $G_{FT3}(V, E)$ is defined as:

$$v_j - v_i \leq \mu(v_i, v_j) \quad \forall i, j = 1, 2, \dots, n$$

where $\mu(v_i, v_j) = \mu(e_{ij}) = \min\{t(e_{ij}), h(e_{ij})\} = w(v_i, v_j) \quad \forall i, j = 1, 2, \dots, n$ where $\mu(v_i, v_j)$ is a membership value of edges e_{ij} , $t(e_{ij})$ is a tail of edges, $h(e_{ij})$ is a head of edges and $w(v_i, v_j)$ is a weight of edges. Next, by using the Definition 2.1, the DBM of FACS is developed and defined as follows:

Definition 3.2: Difference Bound Matrix (DBM) of FACS

The matrix $M_{D_{ij}}^{n \times n}$ associated with potential constraint set C is called a Difference Bound Matrix (DBM) of FACS and is defined as follows:

$$M_{D_{ij}}^{n \times n} = [d_{ij}] \text{ where } d_{ij} = \begin{cases} m_{ij}^t = \mu(v_i, v_j) & \text{for } i \neq j \\ +\infty & \text{for } i = j \end{cases}$$

From the Definition 3.2 and by looking at the example of adjacency matrix of FACS previously, the following Theorem 3.1 is developed.

Theorem 3.1: Every transpose of adjacency matrix of no loop of FACS is a Difference Bound Matrix (DBM)

Proof:

Suppose $G_{FT3}(V, E)$ be a no loop of fuzzy graph which is autocatalytic that is FACS, therefore $G_{FT3}(V, E)$ can be represented by adjacency matrix which is defined as

$$M_{F_{ij}} = [m_{ij}] \text{ where } m_{ij} = \begin{cases} 0, & \text{when } i = j \text{ and } (v_i, v_j) \notin E \\ \mu(e_{ij}) \in (0, 1], & \text{when } i \neq j \text{ and } (v_i, v_j) \in E \end{cases}$$

In other word, $M_{F_{ij}}$ is the adjacency matrix of a directed graph $G_{FT3}(V, E)$ where V is a set of vertices and $E = V \times V = V^2$ is a set of edges. Since it is an adjacency matrix, it can be written as:

$$M_{D_{ij}}^{n \times n} = [d_{ij}] \text{ where } d_{ij} = \begin{cases} +\infty, & \text{for } i = j \\ m_{ij}^t = \mu(v_i, v_j), & \text{for } i \neq j \end{cases}$$

$$\text{and } m_{ij}^t = \mu(v_i, v_j) = \mu(e_{ij}) = \min\{t(e_{ij}), h(e_{ij})\} = w(v_i, v_j)$$

One can replace 0 in $M_{F_{ij}}$ by $+\infty$, since there is no loop in fuzzy graph type-3. Therefore, $M_{D_{ij}}^{n \times n}$ can be seen as Difference Bound Matrix (DBM). By using the Definition 2.3, the potential graph of FACS is defined as follows:

Definition 3.3: Potential Graph of FACS

Let $M_{D_{ij}}^{n \times n}$ be a DBM of a directed graph $\mathcal{G} = (V, A, w)$ with edges weighted in \mathbb{I} . Let V be the set of nodes, $A \subseteq V \times V = V^2$ is the set of edges and $w: A \mapsto \mathbb{I}$ is the weight function for graph \mathcal{G} . Therefore, \mathcal{G} is defined as:

$$\mathcal{G} = \begin{cases} (v_i, v_j) \notin A, & \text{if } d_{ij} = +\infty \\ (v_i, v_j) \in A \text{ and } w(v_i, v_j) = d_{ij}, & \text{if } d_{ij} \neq +\infty \end{cases}$$

Next, the following Theorem 3.2 shows that DBM of FACS is a bijective function.

Theorem 3.2:

Let $D_{k_{FT3}}$ be a Difference Bound Matrix (DBM) of FACS which is autocatalytic that is FACS defined by:

$$D_{k_{FT3}} = \begin{cases} d_{ij} = \mu(v_i, v_j) & \text{for } i \neq j \\ +\infty & \text{for } i = j \end{cases}$$

Let $D_{FT3} = \{D_{k_{FT3}}; k = 1, 2, \dots, n\}$ be a finite set of all DBM of FACS of fuzzy graph type-3. Let $\mathcal{G}(V, A, w) = \{\mathcal{G}_k | k: 1, 2, \dots, n\}$ where

$$\mathcal{G}_k(V, A, w) = \begin{cases} (v_i, v_j) \notin A, & \text{if } d_{ij} = +\infty \\ (v_i, v_j) \in A \text{ and } w(v_i, v_j) = d_{ij}, & \text{if } d_{ij} \neq +\infty \end{cases}$$

Define $\beta: D_{FT3} \rightarrow \mathcal{G} \ni \beta(D_{k_{FT3}}) = \mathcal{G}$ where $d_{ij} = w(v_i, v_j)$. Then, β is a bijective function.

Proof:

1) Let $D_{FT3} = D'_{FT3}$

$$\Rightarrow d_{ij} = d'_{ij}$$

$$\Rightarrow m_{ij}^t = m'_{ij}{}^t$$

$$\Rightarrow w(v_i, v_j) = w'(v'_i, v'_j)$$

$$\Rightarrow \mathcal{G} = \mathcal{G}'$$

$\therefore \beta$ is a well-defined function.

2) $f: A \rightarrow B$ is onto if $b \in B$, then $\exists a \in A \ni f(a) = b$. Thus, \mathcal{G}_{FT3} will be picked, then $\exists [d_{ij}] \in M_{D_{ij}}^{n \times n} \ni \beta[d_{ij}] = \mathcal{G}_{FT3}$ and $[d_{ij}] = w(v_i, v_j)$ for $(v_i, v_j) \in A$ for $\mathcal{G}(V, A, w)$.

$\therefore \beta$ is onto.

3) In the case of $\beta[D_{1_{ij}}] = \beta[D_{2_{ij}}]$

$$\Rightarrow \mathcal{G}_1(V, A, w) = \mathcal{G}'_2(V', A', w')$$

$$\begin{aligned}\Rightarrow V &= \{v_1, v_2, \dots, v_n\} = \{v'_1, v'_2, \dots, v'_n\} = V' \\ \Rightarrow A &= \{\mu(v_i, v_j)\}_{i,j=1,2,\dots,n} = \{\mu'(v'_i, v'_j)\}_{i,j=1,2,\dots,n} = A' \\ \Rightarrow w &= \{w(v_i, v_j)\}_{i,j=1,2,\dots,n} = \{w'(v'_i, v'_j)\}_{i,j=1,2,\dots,n} = w' \\ \Rightarrow D_{1_{FT3}} &= D_{2_{FT3}}\end{aligned}$$

$\therefore \beta$ is one-to-one function.

Therefore, β is a bijective function. Thus, a square matrix can be map to any loop of FACS.

IMPLEMENTATION AND DISCUSSION

All the Definitions mentioned previously are applied in transforming FACS of the clinical waste incineration process. It is started by looking at the Figure 1, values in the matrix, M_F and Definition 3.1, which leads to the following potential constraints.

$$\begin{array}{ll}v_2 - v_1 \leq 0.00001 & v_1 - v_3 \leq 0.06529 \\v_3 - v_1 \leq 0.15615 & v_4 - v_3 \leq 0.63563 \\v_4 - v_1 \leq 0.51632 & v_5 - v_3 \leq 0.00002 \\v_5 - v_1 \leq 0.00001 & v_6 - v_3 \leq 0.29906 \\v_6 - v_1 \leq 0.32752 & v_6 - v_4 \leq 0.00001 \\v_4 - v_2 \leq 0.68004 & v_4 - v_5 \leq 0.99999 \\v_5 - v_2 \leq 0.00001 & v_1 - v_6 \leq 0.13401 \\v_6 - v_2 \leq 0.31995 & \end{array}$$

By using the Definition 3.2 and Theorem 3.1, DBM of FACS for the clinical waste incineration process is presented as follows:

	v_1	v_2	v_3	v_4	v_5	v_6
v_1						
v_2	$+\infty$	0.00001	0.15615	0.51632	0.00001	0.32752
v_3	0.06529	$+\infty$	$+\infty$	0.63563	0.00002	0.29906
v_4	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0.00001
v_5	$+\infty$	$+\infty$	$+\infty$	0.99999	$+\infty$	$+\infty$
v_6	0.13401	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$

By using Definition 3.3, Theorem 3.2 and 3.4, potential graph of FACS is obtained as in Figure3. Here, we let $\theta_{ij} = \mu(e_{ij}) = \mu(v_i, v_j) \in (0,1]$. A function $l: E \rightarrow R^+$ such that $l(\theta_{ij}) = \theta_{ij} \in R^+$ represents the length of the edge $e_{ij} = (v_i, v_j)$ (Noor Aini and Sumarni, 2017) is therefore converted the fuzzy graph in Figure 1 into a crisp graph as in Figure 3. Next, the following Theorem 3.3 is obtained.

Theorem 3.3:

Every no loop of FACS of fuzzy graph type-3 is a potential graph.

Proof:

Suppose that a directed graph $G_{FT3}(V, E)$ of no loop of FACS type-3 is defined as follows:

$$G_{FT3} = \begin{cases} (v_i, v_j) \in E, & \text{if } m_{ij} \neq 0 \\ (v_i, v_j) \notin E, & \text{if } m_{ij} = 0 \end{cases}$$

Since G_{FT3} is a directed graph with edges weighted between $(0,1]$ where $(0,1] \cong \mathbb{R} = \mathbb{I}$, then it can be represented as the following:

$$\mathcal{G} = \begin{cases} (v_i, v_j) \notin A, & \text{if } d_{ij} = +\infty \\ (v_i, v_j) \in A \text{ and } w(v_i, v_j) = d_{ij}, & \text{if } d_{ij} \neq +\infty \end{cases}$$

where d_{ij} is Difference Bound Matrix (DBM) of FACS such that

$$d_{ij} = \begin{cases} +\infty, & \text{for } i = j \\ m_{ij}^t = \mu(v_i, v_j), & \text{for } i \neq j \end{cases}$$

Hence, G_{FT3} can be seen as potential graph.

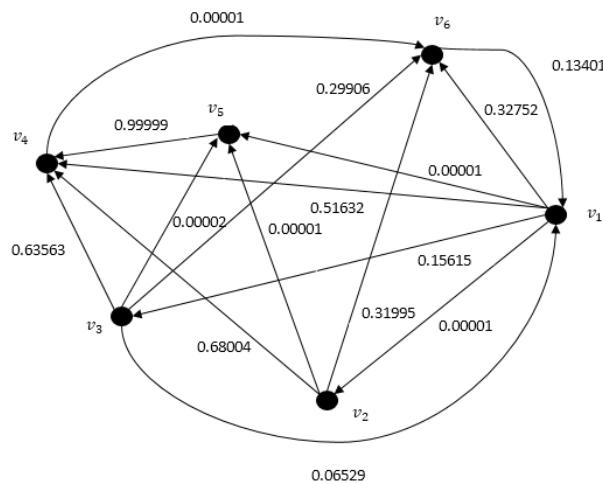


Figure 3. Potential graph of FACS

From the graph in Figure 3, it can be seen that the graph preserved the number of nodes and its corresponding directed edges together with its value. In addition, one can see that the potential graph of FACS seem to be similar to Figure 1 in term of number of connectivity and vertices.

However, the edges have no colours which implies that the graph is no longer a fuzzy graph, but crisp graph. Furthermore, by looking at the DBM of FACS, basic characteristics of the matrix particularly for clinical waste incineration process are deduced and are listed as follows:

- It is non-symmetrical matrix.
- All element of the matrix, $d_{ij} \in (0,1] + \{+\infty\}$.
- At least one entry of each column in the matrix is not $+\infty$.
- The DBM of FACS is the transpose of adjacency matrix of FACS but if $m_{ij} = 0$ in adjacency matrix, then $d_{ij} = +\infty$ in DBM.
- It is $+\infty$ instead of 0 because the variables are insignificantly reacted to each other and remain as it is along the process.
- All the diagonal entries, $d_{ii} = +\infty$.

Analysis on both of the graphs and the characteristic of the DBM of FACS leads to the idea of transforming the FACS to a potential graph of FACS as listed in the following algorithm.

Input: FACS graph

Output: Potential graph of FACS

Begin

- Read input matrix of FACS
- Find potential constraint, $v_j - v_i \leq \mu(v_i, v_j)$ for all $i, j = 1, 2, \dots, n$.
- Obtain the DBM as in Definition 3.2
- Draw the corresponding potential graph of FACS

End

Next, the following theorems and corollaries are deduced.

Theorem 3.4:

Any FACS can be induced to potential graph of FACS.

Proof:

Suppose $G_{FT3}(V, E)$ is an FACS with n vertices. It is primitive, irreducible and aperiodic which fulfils Theorem 2.4, 2.5 and 2.6. Then, define the potential constraints for FACS as in Definition 3.1. Next, the Difference Bound Matrix (DBM) of FACS can be obtained by using Definition 3.2. Subsequently, the potential graph of FACS is obtained by Definition 3.3 which fulfils the conditions in Theorem 3.2 and Theorem 3.3. Finally, FACS is induced to potential graph of FACS of fuzzy graph type-3.

Next, the following corollary is immediate case of Theorem 3.4.

Corollary 3.1

FACS of the clinical waste incineration process can be induced to potential graph of FACS.

Theorem 3.5:

Let $G_{FT3}(V, E)$ be a FACS with $V = \{v_1, v_2, \dots, v_n\}$ and $E = \{e_{ij} = (v_i, v_j) | i, j = 1, 2, \dots, n\}$ and $\mathcal{G}'(V', A', w')$ be the potential graph of FACS. Then, $G_{FT3}(V, E)$ is isomorphism to $\mathcal{G}'(V', A', w')$.

Proof:

Let $G_{FT3}(V, E)$ be a FACS with $V = \{v_1, v_2, \dots, v_n\}$ and $E = \{e_{ij} = (v_i, v_j) | i, j = 1, 2, \dots, n\}$ and $\mathcal{G}'(V', A', w')$ be the potential graph of FACS; i.e. after transformation.

Define $h: V \rightarrow V' \ni h(v_i) = v'_i, \forall i = 1, 2, \dots, n$.

Therefore, $\sigma(v_i) = \sigma'(h(v_i)) = \sigma'(v'_i) = 1, \forall i = 1, 2, \dots, n$ since the transformation preserves the nodes. Furthermore, by taking

$$\mu'(v_i, v_j) = \min\{1, \mu(v_i, v_j)\}, \forall (v_i, v_j) \in E'$$

$$\mu(v_i, v_j) = \mu'(v_i, v_j)$$

Therefore, by Theorem 2.13, $G_{FT3}(V, E)$ is homomorphism to $\mathcal{G}'(V', A', w')$.

Now, let $v'_k \in V'$, then $\exists v_k \in V \ni h(v_k) = v'_k$. Thus, h is onto.

Next, suppose that $h(v_a) = h(v_b)$, therefore

$$v'_a = v'_b \text{ by the definition of } h.$$

$$v_a = v_b \text{ since } |V| = |V'|$$

Thus, h is one to one. If that the case, it is bijective homomorphism that satisfies $\sigma(v_i) = \sigma'(h(v_i)) \forall v_i \in E'$ and $\mu(v_i, v_j) = \mu'(h(v_i), h(v_j)) \forall v_i, v_j \in E'$.

Therefore, $G_{FT3}(V, E)$ is isomorphism to $\mathcal{G}'(V', A', w')$. We denote that $G_{FT3}(V, E) \cong \mathcal{G}'(V', A', w')$.

The following corollary is immediate case of Theorem 3.5.

Corollary 3.2

FACS of clinical waste incineration process, $G_{FT3}(V, E)$ is isomorphism to the potential graph of FACS.

CONCLUSION

This paper proposed different type of matrix representation of FACS by investigating DBM. It is found that DBM of FACS is a bijective function and the graph of FACS can be seen as a potential graph. Furthermore, FACS is also isomorphic to its potential graph of FACS. This fundamental investigation would serve as a platform for further analysis on the properties of the DBM of FACS in explaining any dynamical system.

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