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Analysis on Training and Bootstrap Error Evaluation with Least Square Fitting on 2D Craniofacial Reconstruction

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ABSTRACT

A model averaging method, namely bootstrap is analyzed with Least Square Fitting (LSF) to compute error on craniofacial reconstruction. The fractured region of a skull is being considered in this paper. Due to bias-variance effect, training error decreases as the degree of polynomial increases resulting in an unpleasant and undesirable curve. Bootstrap method is used to better estimate the error. The reliability of the bootstrap method is demonstrated corresponding to a subjective assessment via observation. Then, bootstrap error on LSF is computed on craniofacial reconstruction. Henceforth, this demonstration of LSF is applied to a craniofacial reconstruction to find the best fit curve that joint the fractured region.

Keywords: Bootstrap, Error, Least Square Fitting, Craniofacial Reconstruction

INTRODUCTION

A best fit curve which is also known as a curve that best fits a series of data points that may be selected subjectively by visual representation. By observing a curve visually using graphical representation, one may not be able to conclude the curve that was chosen is a best fit curve. This is because each individual may have different opinion of how the original data looks like before it is contaminated with noise.

In real life, most measured data are prone to noise, missing data, redundancy and outlier problems. Research has been carried out to deal with these types of data. In order to reconstruct a 3 dimensional (3D) mesh model from a set of unorganized, noisy data points, a statistical approach namely Bayesian model is proposed by Qian *et al.* (2006). Based on their research, their experimental results show that their method can be used in removal of outliers, to smooth noisy data, reconstruction of mesh and enhancement of features. Jenke *et al.* (2006) applied similar approach to reconstruct piecewise-smooth surfaces with sharp features and compare the performance of the algorithm using synthetic and real world data sets. In applying these two Bayesian models, a user is requested to input the level of noise. The statistical method is adopted to handle issues related to the stochastic nature of noise. In dealing with scanned point set, Yoon, Ivrissimtzis and Lee (2009) proposed Variational Bayesian (VB) method for noise estimation of 3D point sets. The noise is normally distributed with zero mean and the variance determines the amount of noise. In the approach of surface reconstruction, Ramli and Ivrissimtzis (2009) proposed a model averaging method used in the statistical problem known as the bootstrap method to estimate the test error of the model which can be used directly to compare between models.

Researchers have analyzed the issues related to diagnose craniofacial reconstruction via different prospect of study. Medical surgeons used MRI, CT scan and X rays to diagnose the fractured region but it is difficult. The presence of computer vision technology has enabled the mathematicians to apply their approach in craniofacial reconstruction. To study more detail about craniofacial reconstructions, researchers can refer to Majeed (2016), Majeed *et al.* (2016), Wuyang *et al.* (2010) and Ramli (2012). In order to estimate the outlook of the fractured joints of a skull, the prompt in joining it using a method that requires low cost and high efficiency have been analyzed. This could enable the surgeons to reconstruct the fractured joints easily.

The difference between the predicted value and the original value is represented as an error. Training error of the model is the average distance between approximation data and the original data. Interpolation of the data by passing through all the training data points yields a training error that equals to zero. Thus, low-quality models result from the minimization of training error. Hence, it is expected that test error also known as generalization error, prediction error or true error is able to measure the quality of a model better. Ramli and Ivrissimtzis (2009) stated that training error is expected to monotonically decrease when the model complexity increases. These researchers applied bootstrap technique to estimate the test error for polynomial fittings of locally parametrized 3D point sets.

In this paper, the model averaging method namely bootstrap error estimation is applied on 2 dimensional (2D) craniofacial reconstructions. In particular, the real life data of craniofacial fracture on head injury of a patient is considered. Relying on training error and visual evaluation may be difficult in some cases. Hence, training error and bootstrap error for different degree of fitting on selected data is evaluated. Bootstrap error is computed to reconstruct the curve which produces the best fit for a specific data set. Coherently, bootstrap demonstrates and gives a best fit curve to fit the fractured region of the part.

METHODOLOGY

In our real life situations, naturally visualized images through observation using human naked eyes seem to look pleasing as well as perfectly smooth in appearance. However, most images or data that are obtained from sources such as scanners, telescope and other devices might be contaminated and disturbed. This is because the data might undergo various disturbances throughout the process of occupying. For instance, inaccurate data collections commonly occur due to the level of precision in instrumental set up or a human error such as errors that occur during the recording process of the data. These sorts of data might produce subjectively and quantitatively undesirable results which can affect the computation process. Nevertheless, from our point of view, these images or data are considered to have zero error. Thus, our aim in this research is to obtain a fitting that emphasizes accuracy and closely match the image. Craniofacial data of a head injured patient is used to evaluate our approach in 2D craniofacial reconstruction to enable the surgeons to reconstruct the fractured region. The data are assessed by training error and bootstrap error in which the exposure of the credibility is analyzed later.

Training Error

The average distance between the original training data and the approximation of the model is defined as the training error of the model. In mathematics, training error can be defined as follows:

$$E_T = \frac{1}{N}\sum_{i=1}^{N} |f(x_i) - y_i| \tag{1}$$

where $f(x_i)$ is the estimated data function, y_i is the sample data value and N is the total number of training data. The predicted model obtained can result in underfit, overfit or may be a best fit curve depending on the model complexity.

Bias Variance Trade-off

In order to extend the explanation on training error, the concept of bias variance trade-off that aids us to clarify the training error reliability concept and complexity of the model is discussed. In this paper, bias variance trade-off is not explained in detail but the effect of this matter is discussed. Model complexity is the ability of the model to adapt to a more complicated data. In geometric modeling, a complex model has many parameters. For instance, when dealing with splines, the model complexity increases as the number of control points are increased (Ramli, 2012). In our case, when polynomial fitting is used, the complexity indicates the order of the polynomial. Subsequently, in polynomial fitting, when the model complexity is increased, training error which is referred in this case as bias will decrease. Thus, the increase in variance implies that the test error that will be explained in the next section will also increase. The reason researchers are not recommended to rely or depend on training error to find the quality of an estimate is due to the fact that the model may overfit. While bias reduces, variance increases and affects test error.

Test Error by Bootstrap Error Estimation

A vast number of application used statistics field in interpreting and making evaluations from the gained data (Efron, 1979). In general, an error is the difference between the original value and the predicted value. There are numerous methods for handling problem related to error and fitting in statistics such as goodness-of-fit test, Kolmogorov-Smirnov test, and others (Hastie, Tibshirani and Friedman, 2009). However, to the best of our knowledge, no one has discussed for 2D problem in term of relating it visually. In this research, a statistical method, namely, bootstrap method has been deployed to evaluate the accuracy of fitting. The focus in dealing with the missing data in real life data using LSF with this statistical approach is concerned in this research. A real life data specifically considers a problem of the fractured skull of a patient with a head injury has been examined to obtain a fitting that best fit the fractured region.

Bootstrap error, E_{BS} is referred to as the average distance between the sample data and the estimated data value obtained from the bootstrap procedure. The formula for the bootstrap error is given by:

$$E_{BS} = \frac{1}{B}\frac{1}{N}\sum_{b=1}^{B}\sum_{i=1}^{N}|f^{*(b)} - y_i| \tag{2}$$

$f^{*(b)}$ - represents the model fit function by the set of training data
B - number of observation
N - number of sample data or simulated data
y_i - original sample data

Application on Real Life Data and Data Analysis

The art of managing questionable phenomenon and occasions are also conducted in statistics. For example, the effectiveness of therapeutics medications, evaluation of ground condition in a tunnel and considerably more are being considered with the connectivity aid of statistics. Statistics are utilized as a part of each field of science.

In this research, the craniofacial fractured region has been chosen and pointed out to present the idea of the bootstrap method as an attribute to nature problem. Researchers have analyzed issues concerning craniofacial reconstruction via different prospects of study. To diagnose fractured regions, surgeons have used Computed Tomography (CT) scan, X-rays, and Magnetic Resonance Imaging (MRI), but it is a challenging process. Nonetheless, the availability of computer vision technology has enabled mathematicians to apply their approaches in 3D craniofacial reconstruction. Fracture of the bones around your eyes, cheekbones, skull, upper or lower jawbone, frontal sinus bone or nasal bone is often been reconstructed using different methods of approach in medical prospects which some requires high cost and results in low efficiency.

The original image of the CT scan slice is adopted from Majeed *et al.* (2015) for comparative analysis. In this paper, the fitting of curve is analyzed at the joints of the fractured part by demonstration using least square method. In Majeed *et al.* (2015), they use normalized mean square error for predicted estimator of the reconstruction.

The original image of the head injury and boundary layer of the head is shown in Figure 1 and Figure 2.

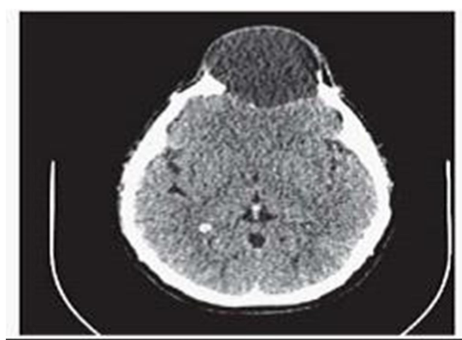

Figure 1: Original image from a patient of head injury

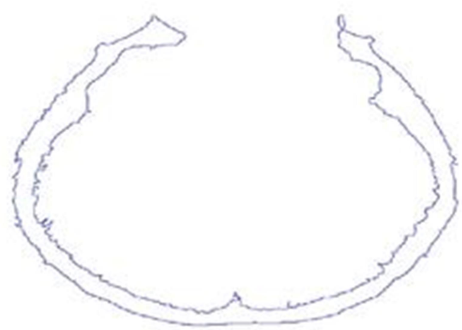

Figure 2: Boundary layer of the head

Selection of Points

The selection of 5 points from the left and 5 points from the right side is deployed to fit the outer layer of the boundary. For the inner layer, similar steps are repeated. The selection of points from the original image for this problem is very crucial in the reconstruction of the fractured joints.

Points that are chosen further away from the fractured region yield a different curve joints. For a suitable choosing of points, the part neighboring the fractured area is considered.

Least Square Fitting (Outer Layer)

In this portion, the reconstruction of polynomial LSF for the outer and inner layer that are linear, quadratic, cubic, quartic, quintic and sextic as shown in Figure 4 and Figure 6 are demonstrated.

From the image on the boundary of the patient's skull, the reconstructions of the fractured parts are analyzed through layer by layer for more precise estimation of the outlook of the joints. Firstly, the fitting of the outer layer of the contour is observed.

Based on Figure 4, the graphical visualisation implies that the degree of fitting that best suits for the new outlook of the fractured joints is the quadratic degree polynomial curve. The selected outer layer curve is shown in Figure 3. The quadratic curve interpolate the points and estimate the fitting of the curve pleasantly. The approximation of the estimated joint curve is reconstructed without knowing the exact contour of the skull. For further evaluation of the curve joints constructed, the bootstrap error estimation is computed to value the credibility of the obtained curve as depicted in Table 1. The compilation of different polynomial degree of fitting is shown in Figure 4.

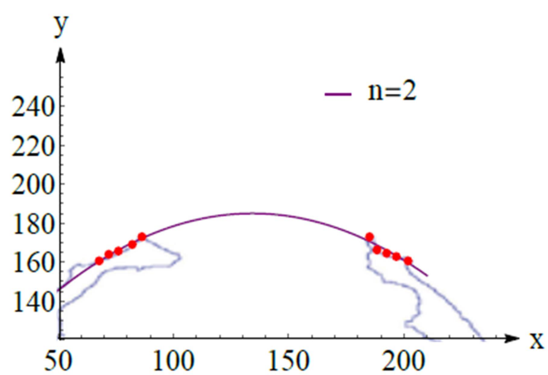

Figure 3: Selected Quadratic Degree Polynomial for Outer Layer

Table 1: The bootstrap error estimation of 100 observations, B = 100 with different polynomial degree for outer layer

Polynomial Degree, n	Training Error, E_T	Bootstrap Error, E_{BS}
Linear ($n = 1$)	3.5213	3.7418
Quadratic ($n = 2$)	1.0227	1.1501
Cubic ($n = 3$)	1.0259	1.8055
Quartic ($n = 4$)	0.7827	1.5583
Quintic ($n = 5$)	0.7047	27.829
Sextic ($n = 6$)	0.4002	1092.1

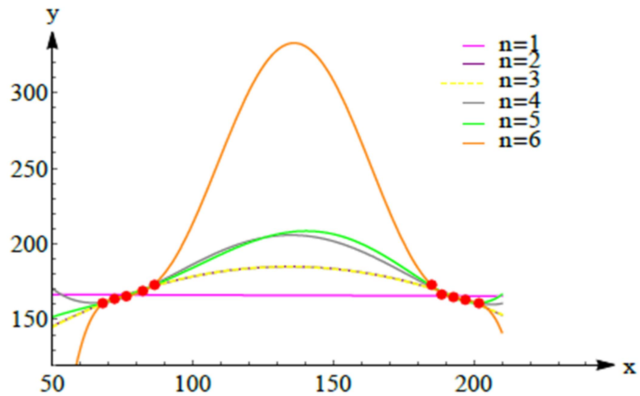

Figure 4: Combined Polynomial Fitting for Outer Layer

Referring to Table 1, the training error decreases as polynomial degree increases. This indicates that for a higher degree polynomial, the curve approximates fitting that tend to be overfit. Conversely, for the outer layer problem, it shows that quadratic degree polynomial is the smallest resulting in a best fit curve. This subsequently in agreement with the graphical visualization showing that quadratic degree suits best the joints of the fractured region of outer layer. Similarly, the following procedure is evaluated to estimate the best curve outlook for inner layer in the next section.

Least Square Fitting (Inner Layer)

From the boundary of the inner layer skull, similar approach is repeated. The graphical visualization shown in Figure 6 for inner layer indicates that as the model complexity increases, the training error becomes smaller. If researcher were to rely on training error itself, they will tend to choose the fitting of degree with smaller error. It seems to produce a joint of the curve that is unpleasant and does not suits the fractured region well. Henceforth, same steps are applied using the adopted statistical method to demonstrate the reliability of the fitting. The fitting of all the degree is compiled in Figure 6. In this case, the visual interpretation shows us that quadratic fits and produce a curve joining the fractured part precisely as shown in Figure 5.

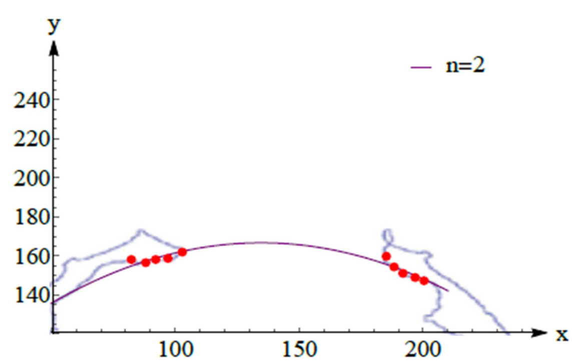

Figure 5: Selected Quadratic Degree Polynomial for Inner Layer

Table 2: The bootstrap error estimation of 100 observations, $B = 100$ with different polynomial degree for inner layer

Polynomial Degree, n	Training Error, E_T	Bootstrap Error, E_{BS}
Linear ($n = 1$)	2.7309	3.0636
Quadratic ($n = 2$)	1.5171	1.7612
Cubic ($n = 3$)	1.1339	2.2683
Quartic ($n = 4$)	0.5369	8.2454
Quintic ($n = 5$)	0.3794	17.589
Sextic ($n = 6$)	0.3789	31.699

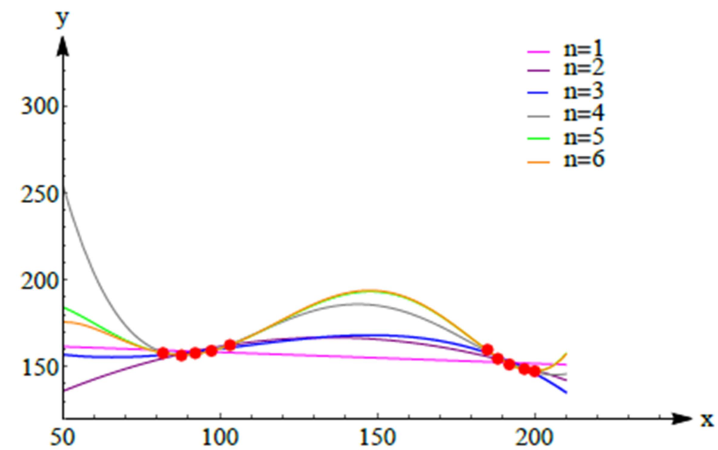

Figure 6: Combined Polynomial Fitting for Inner Layer

According to the computed error that was tabulated in Table 2, the values of training error decreases down the degree. Bootstrap error shows the lowest error coherently same with the outer layer. Conclusively, the least error in the quadratic degree indicates a best fit curve joining the fractured region. These both layer closely mimics the graphical visualisation.

Craniofacial Reconstruction of the Fractured Region

The reconstruction of craniofacial fracture region of the patient with head injury is evaluated using LSF and shown in Figure 7

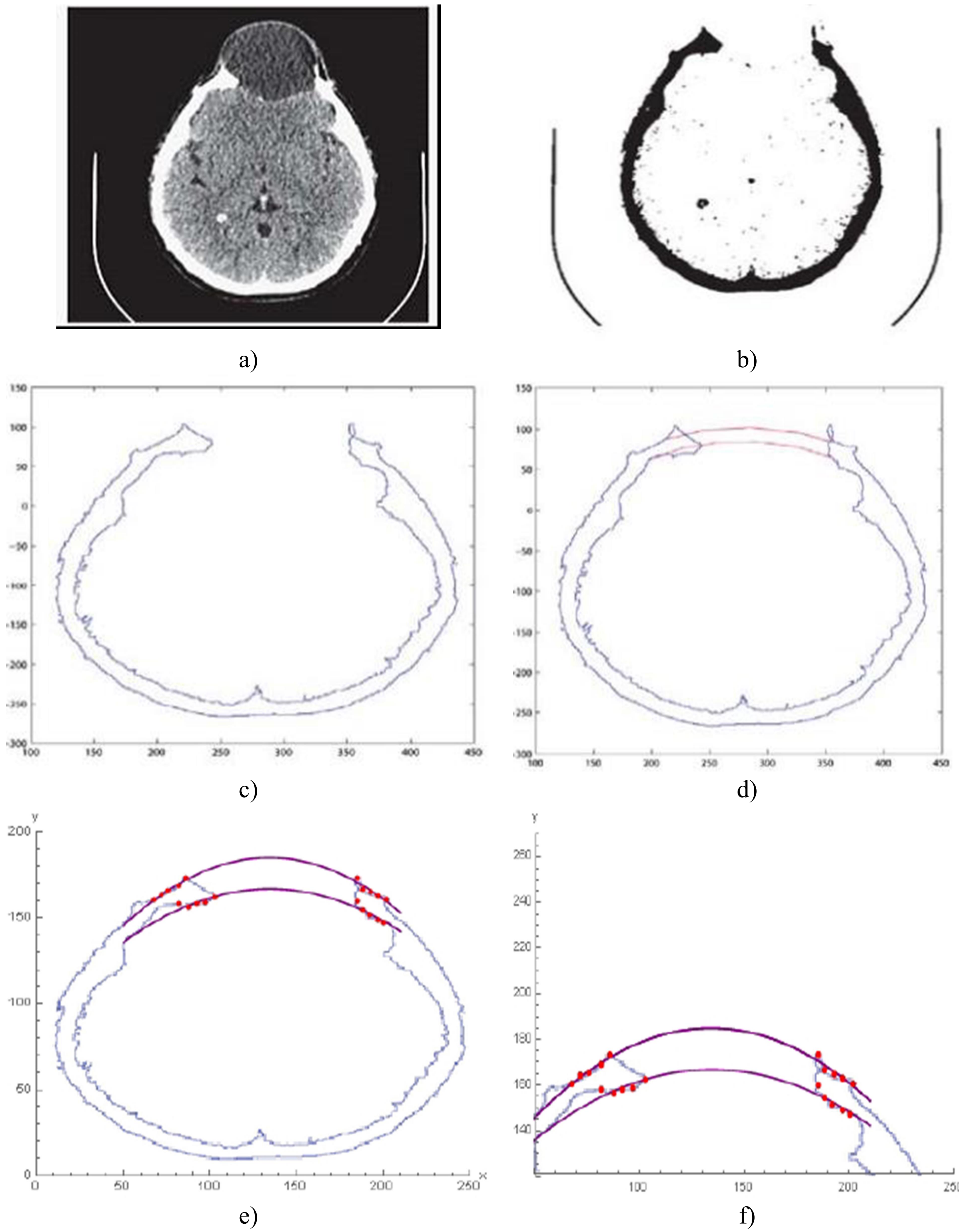

Figure 7: Craniofacial Reconstruction of the Fractured Region
a) Original image (b) Binary image (c) Boundary (d) Reconstructed boundary using rational cubic ball (e) Reconstructed boundary using LSF

Original image, boundary image and reconstructed boundary using rational cubic ball image are obtained from Majeed *et al.* (2015). As observed from Figure 7, the reconstructed boundary using rational cubic ball by Majeed *et al.* (2015) is attach together for comparison purposes. As concerned, the exact contour of the patient with head injury is unidentified and unknown.

Therefore, by estimating a new outlook to predict the joints, bootstrap error method that emphasizes on simplicity is applied to enhance the surgeons to diagnose. In this case, the fitting of the joints constructed best suits the fractured region. The curve is reconstructed using the points including points from the ends of the fractured part at both ends. In this section, the curve constructed by Majeed (2016) where the joints do not connect to the end points of the fractured region is observed. They used rational cubic ball to reconstruct the curve. Both approach of approximating the joints come from different prospect and method. In this research, LSF using bootstrap method is applied on 2D craniofacial reconstruction to enhance the surgeons for more simplified version in handling this problem. From our application, the outlook of the joints observed seems to match and joints the fractured part precisely. Bootstrap error estimation can be used to better estimate the curve for the craniofacial reconstruction.

CONCLUSION

The best model representation depends on the quality of fitting obtained. Training error itself is unable to identify the best model. In this paper, bootstrap error on 2D noisy data is discussed and a comparative discussion has been made based on a visual observation and computed errors. This approach is applied using a real life data of a patient with head injury to better estimate the craniofacial reconstruction. The bootstrap error estimation is chosen to rely on rather than training error since the quality of fitting produced by bootstrap error estimation is better compared to training error. This discussion is believed will assist other researchers in seeing the benefit of bootstrap error.

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