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Co-prime Probability for Nonabelian Metabelian Groups of Order 24 and Their Related Graphs

Nurfarah Zulkifli[1] and Nor Muhainiah Mohd Ali[2]
[1,2]*Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 Johor Bahru, Johor*
[1]nurfarahzulkifli@yahoo.com, [2]normuhainiah@utm.my

ABSTRACT

Let G be a finite group. The probability of a random pair of elements in G are said to be co-prime when the greatest common divisor of order x and y, where x and y in G is equal to one. Meanwhile the co-prime graph of a group is defined as a graph whose vertices are elements of G and two distinct vertices are adjacent if and only if the greatest common divisor of order x and y is equal to one. The study of the co-prime probability and the co-prime graph with different kinds of groups have been widely spread among the researchers for the last few years. Unfortunately, none did a research on both the co-prime probability and its graphs. Hence, this research focuses on the co-prime probability and its graphs for nonabelian metabelian groups of order 24. The research starts by calculating the co-prime probability and later, constructing the co-prime graph. Later, the types of graph, the number of edges, the dominating number and the independent number are obtained. The definitions, theorems and propositions by previous researches are used to achieve the objectives of this research.

Keywords: Co-prime probability, Co-prime-graph, Dominating number, Independent number

INTRODUCTION

A simple explanation on probability is how likely something is to happen. For an example, after flipping a coin, what is the probability or possibility that the coin will land on its tail? The probability theory is very much helpful for making prediction or decision which also helps in research investigation so that further analysis can be made. Probability is often not just being used by statisticians to solve any uncertainties but it has also been applied in the field of group theory. Mathematically, the probability of an event to happen is number of ways it happens divided by total number of outcomes.

The study of the probability that two elements in a group commute, denoted by $P(G)$ has been widely spread among mathematicians since 1968 by Erdos and Turan (1968) whom the first to discover about it. Later, researchers began to dig deeper into the commutativity degree using nonabelian metabelian groups and some extension has been done such as the n^{th} commutativity degree, the multiplicative degree, the sub-multiplicative degree, the relative commutativity degree and many more. Abd Rhani (2018) then extended the research of the commutativity degree of G to the co-prime probability of G. This probability is defined as the probability of the order of a random pair of elements in the group are relatively prime or co-prime. In her research, the study was focused on all $p-$groups and for some dihedral groups.

The extension of the prime graph started in 1981 where Williams (1981) introduced the prime graph of G and is defined as the prime numbers dividing the order of G and two vertices u and v are joined by an edge if and only if G contains an element of order uv. Subsequently, the study began to develop and the extension of the prime graph has been extensively explored using different types of groups. Erdos and Sarkozy (1997) introduced the co-prime graph of integers and the cycles of the co-prime graph are determined. Later, Sarkozy (1999) continued the study of the co-prime graph of integers and, in his study, the complete tripartite subgraphs were obtained. Then, Sattanathan and Kala (2009) studied on the order prime graph and in their research, certain properties of order prime graph, lower bounds and upper bounds on the number of edges of order prime graphs and the character for certain classes were attained. Next, Ma et al. (2014) extended the research by introducing the co-prime graph for finite group. The types and some properties of the graphs were then determined. Unfortunately, none did a research on both the co-prime probability and its graphs for nonabelian metabelian groups of order 24.

Therefore, in this research, the co-prime probability and its graphs together with the number of edges, the types of graph, the dominating number and the independent number for nonabelian metabelian groups of order 24 are determined.

Hence, this research is organized as follows: the first part explains the introduction of the research follows by the second part which states the basic concepts, definitions, propositions, and theorems that are useful for this research. Next, the main results of the co-prime probability and its graphs for nonabelian metabelian groups of order 24 are explained before the conclusion is made on the last part of this research.

PRELIMINARIES ON GROUPS AND GRAPHS

In this section, some basic concept on group and graph theory that will be used in this research are stated.

Definition 1 (Wisnesky, 2005) A group G is metabelian if there exists a normal subgroup A such that A and G/A are abelian.

Definition 2 (Abd Rhani, 2018) Let G be a finite group. For any $x, y \in G$, the co-prime probability of G, denoted as $P_{copr}(G)$ is defined as:

$$P_{copr}(G) = \frac{\left|\left\{(x, y) \in G \times G, (|x|, |y|) = 1\right\}\right|}{|G|^2}.$$

In this research, $(|x|, |y|)$ indicates the greatest common divisor of x and y, where $x, y \in G$.

Definition 3 (Bondy and Murty, 2008) A graph Γ is connected if each pair of the vertices are joined by a path.

Definition 4 (Bondy and Murty, 2008) The degree of vertex x, $\deg(x)$ is the number of edges incident with x, each loop counting as two edges.

Definition 5 (Bondy and Murty, 2008) The dominating set $X \subseteq V(\Gamma)$ is a set where for each v outside W there exist $x \in X$ such that v is adjacent to x. The minimum size of X is called the dominating number and it is denoted by $\gamma(\Gamma)$.

Definition 6 (Bondy and Murty, 2008) A non-empty S of $V(\Gamma)$ is called an independent set of Γ if there is no adjacent between two elements of S in Γ. Thus the independent number is the number of vertices in maximum independent set and it is denoted as $\alpha(\Gamma)$.

Definition 7 (Godsil and Royle, 2001) A bipartite graph or a bigraph, $K_{m,n}$ is a set of vertices partitioned in two subsets such that there is no adjacent between two graph vertices within the same set.

Definition 8 (Ma et al., 2014) The co-prime graph of G denoted as $\Gamma_{copr}(G)$ is a graph whose vertices are elements of G and two distinct vertices u and v are adjacent if and only if $(|x|,|y|)=1$.

Proposition 1 (Ma et al., 2014)
Let G be a group. Then, G is not a $p-\text{groups}$ if and only if $\Gamma_{copr}(G)$ is not bipartite.

Proposition 2 (Ma et al., 2014)
Let G be a group with order greater than 2. Then, $\{e\}$ is unique dominating set of size 1 of $\Gamma_{copr}(G)$ In particular, $\gamma(\Gamma_{copr}(G))=1$ and $\deg_{\Gamma_{copr}(G)}(e)=|G|-1$.

CO-PRIME PROBABILITY WITH THEIR RELATED GRAPHS FOR NONABELIAN METABELIAN GROUPS OF ORDER 24

The first part of this section gives the co-prime probability for nonabelian metabelian groups of order 24, while the second part of this section discusses the construction and the determination of the co-prime graph together with the types of graph, the number of edges, the dominating number and the independent number.

Co-Prime Probability for Nonabelian Metabelian Groups of Order 24

In this subsection, the co-prime probability for nonabelian metabelian groups of order 24 are resolved. Firstly, the order of each element in the group is determined. Then, by using Definition 8, the co-prime probability for nonabelian metabelian groups of order 24 are specified. The following theorem gives the finding of the coprime probability for nonabelian metabelian groups of order 24.

Theorem 1

Let G be nonabelian metabelian group of order 24. Then,

$$P_{copr}(G)=\begin{cases} 107/576, & \text{if } G = S_3\times\mathbb{Z}_4, S_3\times\mathbb{Z}_2\times\mathbb{Z}_2, (\mathbb{Z}_6\times\mathbb{Z}_2)\rtimes\mathbb{Z}_2 \text{ and } \mathbb{Z}_3\rtimes Q,\\ 75/576, & \text{if } G = D_4\times\mathbb{Z}_3 \text{ and } Q\times\mathbb{Z}_3,\\ 131/576, & \text{if } G=\mathbb{Z}_2\times\mathbb{Z}_3\times\mathbb{Z}_4 \text{ and } \mathbb{Z}_3\times\mathbb{Z}_8,\\ 159/576, & \text{if } G = A_4\times\mathbb{Z}_2,\\ 99/576, & \text{if } G = D_{12}. \end{cases}$$

Proof

Let $G=S_3\times\mathbb{Z}_4$ where $G=\langle a,b,c \mid a^3=b^2=c^4=abab=aca^{-1}c^{-1}=bcb^{-1}c^{-1}=1\rangle$ = $\{e,a,a^2,b,c$ $c^2,c^3,ab,ac,ac^2,ac^3,a^2b,a^2c,a^2c^2,a^2c^3,bc,bc^2,bc^3,abc,abc^2,abc^3,a^2bc,a^2bc^2,a^2bc^3\}$. Here, it can be found that $|e|=1$, $|b|=|c^2|=|ab|=|a^2b|=|bc^2|=|abc^2|=|a^2bc^2|=2$, $|a|=|a^2|=3$, $|c|=|c^3|=|bc|=|bc^3|=|abc|=|abc^3|=|a^2bc|=|a^2bc^3|=4$, $|ac^2|=|a^2c^2|=6$ and $|ac|=|ac^3|=|a^2c|=|a^2c^3|=12$. In order to find $P_{copr}(G)$, three cases should be considered as follows:

Case 1: If $x=e$, then x is co-prime to each element y in G since $(|x|,|y|)=(|e|,|y|)=1$. Let $N_1=\{(e,y)\in G\times G\}$. Then, $|N_1|=24$. Now, if $y=e$, then y is also co-prime to all x in $G\setminus e$ since $(|x|,|y|)=(|x|,|e|)=1$. Let $|N_2|=\{(x,e)\in G\setminus e\times G\}$. Then, $|N_2|=23$.

Case 2: Let $N_3=\{(x,y)\in G\times G \| x|=2,|y|=3\}$, then $x\in\{b,c^2,ab,a^2b,bc^2,abc^2,a^2bc^2\}$ and $y\in\{a,a^2\}$. This implies $|N_3|=14$. Next, let $N_4=\{(x,y)\in G\times G \| x|=3,|y|=2\}$, then $x\in\{a,a^2\}$ and $y\in\{b,c^2,ab,a^2b,bc^2,abc^2,a^2bc^2\}$. Thus, $|N_4|=14$.

Case 3: Let $N_5=\{(x,y)\in G\times G \| x|=4,|y|=3\}$, then $x\in\{c,c^3,bc,bc^3,abc,abc^3,a^2bc,a^2bc^3\}$ and $y\in\{a,a^2\}$. This implies $|N_5|=16$. Next, let $N_6=\{(x,y)\in G\times G \| x|=3,|y|=4\}$, then $x\in\{a,a^2\}$ and $y\in\{c,c^3,bc,bc^3,abc,abc^3,a^2bc,a^2bc^3\}$. Thus, $|N_6|=16$.

Hence, $P_{copr}(S_3\times\mathbb{Z}_4)=\dfrac{|N_1+N_2+N_3+N_4+N_5+N_6|}{|S_3\times\mathbb{Z}_4|^2}=\dfrac{107}{576}$.

Next, let $G=S_3\times\mathbb{Z}_2\times\mathbb{Z}_2$ where $G=\langle a,b,c \mid a^6=b^2=c^2=abab=aca^{-1}c^{-1}=bcb^{-1}c^{-1}=1\rangle$ = $\{e,a,a^2,a^3,a^4,a^5,b,c,ab,a^2b,a^3b,a^4b,a^5b,ac,a^2c,a^3c,a^4c,a^5c,bc,abc,a^2bc,a^3bc,a^4bc,a^5bc\}$. It can be found that $|e|=1$, $|a|=|b|=|c|=|ab|=|a^3b|=|a^4b|=|a^5b|=|a^3c|=|bc|=|abc|=|a^2bc|=|a^3bc|=|a^4bc|=|a^5bc|=2$, $|a|=|a^4|=3$ and $|a|=|a^5|=|a^2b|=|ac|=|a^2c|=|a^4c|=|a^5c|=6$. Two cases below should be considered.

Case 1: If $x = e$, then x is co-prime to each element y in G since $(|x|,|y|) = (|e|,|y|) = 1$. Let $N_1 = \{(e, y) \in G \times G\}$. Then, $|N_1| = 24$. Now, if $y = e$, then y is also co-prime to all x in $G \setminus e$ since $(|x|,|y|) = (|x|,|e|) = 1$. Let $N_2 = \{(x, e) \in G \setminus e \times G\}$. Then, $|N_2| = 23$.

Case 2: Let $N_3 = \{(x, y) \in G \times G \| x| = 2, |y| = 3\}$, then $x \in \{a^3, b, c, ab, a^3b, a^4b, a^5b, a^3c, bc, abc, a^2bc, a^3bc, a^4bc, a^5bc\}$ and $y \in \{a^2, a^4\}$. This implies $|N_3| = 30$. Next, let $N_4 = \{(x, y) \in G \times G| |x| = 3, |y| = 2\}$, then $x \in \{a^2, a^4\}$ and $y \in \{a^3, b, c, ab, a^3b, a^4b, a^5b, a^3c, bc, abc, a^2bc, a^3bc, a^4bc\ a^5bc\}$. Thus, $|N_4| = 30$.

Hence, $P_{copr}(S_3 \times \Box_2 \times \Box_2) = \dfrac{|N_1 + N_2 + N_3 + N_4|}{|S_3 \times \Box_2 \times \Box_2|^2} = \dfrac{107}{576}$.

Let $G = (\Box_6 \times \Box_2)\text{ã}\ \Box_2$ where $G = \langle a, b, c \mid a^3 = b^2 = c^2 = (cb)^4 = 1, ab = ba, aca = c\rangle = \{e, a, a^2, b, c, ab, a^2b, ac, a^2c, abc, a^2bc, bc, bcbc, cb, acb, a^2cb, bcb, abcb, a^2bcb, cbc, acbc, a^2cbc, abcbc, a^2bcbc\}$. It is found that $|e| = 1$, $|b| = |c| = |ac| = |a^2c| = |bcbc| = |bcb| = |abcb| = |cbc| = |a^2bcb| = 2$, $|a| = |a^2| = 3$, $|abc| = |a^2bc| = |bc| = |cb| = |acb| = |a^2cb| = 4$ and $|a^2cbc| = |a^2bcbc| = |abcb| = |ab| = |a^2b| = |acbc| = 6$. Below are the cases that should be considered to find $P_{copr}(G)$.

Case 1: If $x = e$, then x is co-prime to each element y in G since $(|x|,|y|) = (|e|,|y|) = 1$. Let $N_1 = \{(e, y) \in G \times G\}$. Then, $|N_1| = 24$. Now, if $y = e$, then y is also co-prime to all x in $G \setminus e$ since $(|x|,|y|) = (|x|,|e|) = 1$. Let $| N_2 |= \{(x, e) \in G \setminus e \times G\}$. Then, $| N_2 |= 23$.

Case 2: Let $N_3 = \{(x, y) \in G \times G \| x| = 2, |y| = 3\}$, then $x \in \{b, c, ac, a^2c, bcbc, bcb, abcb, a^2bcb, cbc\}$ and $y \in \{a, a^2\}$. This implies $|N_3| = 18$. Next, let $N_3 = \{(x, y) \in G \times G \| x| = 3, |y| = 2\}$, then $x \in \{a, a^2\}$ and $y \in \{b, c, ac, a^2c, bcbc, bcb, abcb, a^2bcb, cbc\}$. Thus, $|N_4| = 18$.

Case 3: Let $N_5 = \{(x, y) \in G \times G \| x| = 4, |y| = 3\}$, then $x \in \{abc, a^2bc, bc, cb, acb, a^2cb\}$ and $y \in \{a, a^2\}$. This implies $|N_5| = 12$. Next, let $N_6 = \{(x, y) \in G \times G \| x| = 3, |y| = 4\}$, then $x \in \{a, a^2\}$ and $y \in \{abc, a^2bc, bc, cb, acb, a^2cb\}$. Thus, $|N_6| = 12$.

Hence, $P_{copr}((\Box_6 \times \Box_2)\text{ã}\ \Box_2) = \dfrac{|N_1 + N_2 + N_3 + N_4 + N_5 + N_6|}{|(\Box_6 \times \Box_2)\text{ã}\ \Box_2|^2} = \dfrac{107}{576}$.

Let $G = \Box_3\text{ã}\ Q$ where $G = \langle a, b \mid a^{12} = b^4 = abab^{-1} = a^6b^2 = 1\rangle = \{e, a, a^2, a^3, a^4, a^5, a^6, a^7, a^8, a^9, a^{10}, a^{11}, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b, a^8b, a^9b, a^{10}b, a^{11}b\}$. It is found that $|e| = 1$, $|a^6| = 2$, $|a^4| = |a^8| = 3$, $|a^3| = |a^9| = |b| = |ab| = |a^2b| = |a^3b| = |a^4b| = |a^5b| = |a^6b| = |a^7b| = |a^8b| = |a^9b| = |a^{10}b| = |a^{11}b| = 4$, $|a^2| = |a^{10}| = 6$ and $|a| = |a^5| = |a^7| = |a^{11}| = 12$. Three cases below should be considered.

Case 1: If $x=e$, then x is co-prime to each element y in G since $(|x|,|y|)=(|e|,|y|)=1$. Let $N_1=\{(e,y)\in G\times G\}$. Then, $|N_1|=24$. Now, if $y=e$, then y is also co-prime to all x in $G\backslash e$ since $(|x|,|y|)=(|x|,|e|)=1$. Let $|N_2|=\{(x,e)\in G\backslash e\times G\}$. Then, $|N_2|=23$.

Case 2: Let $N_3=\{(x,y)\in G\times G \| x|=2,|y|=3\}$, then $x=a^6$ and $y\in\{a^4,a^8\}$. This implies $|N_3|=18$. Next, let $N_4=\{(x,y)\in G\times G \| x|=3,|y|=2\}$, then $x\in\{a^4,a^8\}$ and $y=a^6$. Thus, $|N_4|=18$.

Case 3: Let $N_5=\{(x,y)\in G\times G \| x|=4,|y|=3\}$, then $x\in\{a^3,a^9,b,ab,a^2b,a^3b,a^4b,a^5b,a^6b,a^7b,$ $a^8b,a^9b,a^{10}b,a^{11}b\}$ and $y\in\{a^4,a^8\}$. This implies $|N_5|=12$. Next, let $N_5=\{(x,y)\in G\times G|$ $|x|=3,|y|=4\}$, then $x\in\{a^4,a^8\}$ and $y\in\{a^3,a^9,b,ab,a^2b,a^3b,a^4b,a^5b,a^6b,a^7b,a^8b,a^9b,a^{10}b,$ $a^{11}b\}$. Thus, $|N_6|=12$.

Hence, $P_{copr}(\mathbb{Z}_3\times Q)=\dfrac{|N_1+N_2+N_3+N_4+N_5+N_6|}{|\mathbb{Z}_3\times Q|^2}=\dfrac{107}{576}$.

Let $G=D_4\times\mathbb{Z}_3$ where $G=\langle a,b,c \mid a^4=b^2=c^3=1, baba=1, ac=ca, bc=cb\rangle=\{e,a,a^2,a^3,b,c$ $c^2,ab,a^2b,a^3b,ac,a^2c,a^3c,ac^2,a^2c^2,a^3c^2,bc,bc^2,abc,a^2bc,a^3bc,abc^2,a^2bc^2,a^3bc^2\}$. It is found that $|e|=1$, $|a^2|=|b|=|ab|=|a^2b|=|a^3b|=2$, $|c|=|c^2|=3$, $|a|=|a^3|=4$, $|a^2c|=|a^2c^2|=|bc|=$ $|bc^2|=|abc|=|a^2bc|=|a^3bc|=|abc^2|=|a^2bc^2|=|a^3bc^2|=6$ and $|ac|=|a^3c|=|ac^2|=|a^3c^2|=12$. Below are the cases that should be considered to find the $P_{copr}(G)$.

Case 1: If $x=e$, then x is co-prime to each element y in G since $(|x|,|y|)=(|e|,|y|)=1$. Let $N_1=\{(e,y)\in G\times G\}$. Then, $|N_1|=24$. Now, if $y=e$, then y is also co-prime to all x in $G\backslash e$ since $(|x|,|y|)=(|x|,|e|)=1$. Let $|N_2|=\{(x,e)\in G\backslash e\times G\}$. Then, $|N_2|=23$.

Case 2: Let $N_3=\{(x,y)\in G\times G \| x|=2,|y|=3\}$, then $x\in\{a^2,b,ab,a^2b,a^3b\}$ and $y\in\{c,c^2\}$. This implies $|N_3|=10$. Next, let $N_4=\{(x,y)\in G\times G \| x|=3,|y|=2\}$, then $x\in\{c,c^2\}$ and $y\in\{a^2,b,$ $ab,a^2b,a^3b\}$. Thus, $|N_4|=10$.

Case 3: Let $N_5=\{(x,y)\in G\times G \| x|=4,|y|=3\}$, then $x\in\{a,a^3\}$ and $y\in\{c,c^2\}$. This implies $|N_5|=4$. Next, let $N_6=\{(x,y)\in G\times G \| x|=3,|y|=4\}$, then $x\in\{c,c^2\}$ and $y\in\{a,a^3\}$. Thus, $|N_6|=4$.

Hence, $P_{copr}(D_4\times\mathbb{Z}_3)=\dfrac{|N_1+N_2+N_3+N_4+N_5+N_6|}{|D_4\times\mathbb{Z}_3|^2}=\dfrac{75}{576}$.

Let $G = Q\times\square_3$ where $G=\langle a,b,c \mid a^3=b^4=c^2=1, b^3c=cb, ab=ba, ac=ca\rangle = \{e,a,a^2,b,b^2,b^3, c,ab,ab^2,ab^3,ac,a^2b,a^2b^2,a^2b^3,a^2c,bc,b^2c,b^3c,abc,ab^2c,ab^3c,a^2bc,a^2b^2c,a^2b^3c\}$. It is found that $|e|=1$, $|b^2|=|c|=|bc|=|b^2c|=|b^3c|=2$, $|a|=|a^2|=3$, $|b|=|b^3|=4$, $|ab^2|=|ac|=|a^2b^2|=|abc|=|a^2b^3c|=|ab^3c|=|a^2bc|=|a^2b^2c|=|a^2c|=|ab^2c|=6$ and $|ab|=|ab^3|=|a^2b|=|a^2b^3|=12$. Below are the cases that should be considered to find the $P_{copr}(G)$.

Case1: If $x=e$, then x is co-prime to each element y in G since $(|x|,|y|)=(|e|,|y|)=1$. Let $N_1=\{(e,y)\in G\times G\}$. Then, $|N_1|=24$. Now, if $y=e$, then y is also co-prime to all x in $G\setminus e$ since $(|x|,|y|)=(|x|,|e|)=1$. Let $|N_2|=\{(x,e)\in G\setminus e\times G\}$. Then, $|N_2|=23$.

Case 2: Let $N_3=\{(x,y)\in G\times G \| x|=2,|y|=3\}$, then $x\in\{b^2,c,bc,b^2c,b^3c\}$ and $y\in\{a,a^2\}$. This implies $|N_3|=10$. Next, let $N_4=\{(x,y)\in G\times G\| x|=3,|y|=2\}$, then $x\in\{a,a^2\}$ and $y\in \{b^2,c,bc,b^2c,b^3c\}$. Thus, $|N_4|=10$.

Case 3: Let $N_5=\{(x,y)\in G\times G\| x|=4,|y|=3\}$, then $x\in\{b,b^3\}$ and $y\in\{a,a^2\}$. This implies $|N_5|=4$. Next, let $N_6=\{(x,y)\in G\times G\| x|=3,|y|=4\}$, then $x\in\{a,a^2\}$ and $y\in\{b,b^3\}$. Thus, $|N_6|=4$.

Hence, $P_{copr}(Q\times\square_3)=\dfrac{|N_1+N_2+N_3+N_4+N_5+N_6|}{|Q\times\square_3|^2}=\dfrac{75}{576}$.

Let $G=\square_2\times\square_3\times\square_4$ where $G=\langle a,b \mid a^4=b^6=1, bab=a\rangle = \{e,a,a^2,a^3,b,b^2,b^3,a^3b^2a,a^3ba, ab,ab^2,ab^3,b^2a,ba,a^2b,a^2b^2,a^2b^3,ab^2a,aba,a^3b,a^3b^2,a^3b^3,a^2b^2a,a^2ba\}$. It is found that $|e|=1$, $|a^2|=|b^3|=|a^2b^3|=2$, $|b^2|=|a^3b^2a|=|a^3b^3|=3$, $|a|=|a^3|=|ab|=|ab^2|=|ab^3|=|b^2a|=|ba|=|a^3b|=|a^2b^3|=|a^3b^2|=|a^2b^2a|=|a^2ba|=4$ and $|b|=|a^3ba|=|a^2b|=|a^2b^2|=|ab^2a|=|aba|=6$. Three cases below should be considered.

Case 1: If $x=e$, then x is co-prime to each element y in G since $(|x|,|y|)=(|e|,|y|)=1$. Let $N_1=\{(e,y)\in G\times G\}$. Then, $|N_1|=24$. Now, if $y=e$, then y is also co-prime to all x in $G\setminus e$ since $(|x|,|y|)=(|x|,|e|)=1$. Let $|N_2|=\{(x,e)\in G\setminus e\times G\}$. Then, $|N_2|=23$.

Case 2: Let $N_3=\{(x,y)\in G\times G\| x|=2,|y|=3\}$, then $x\in\{a^2,b^3,a^2b^3\}$ and $y\in\{b^2,a^3b^2a,a^3b^3\}$. This implies $|N_3|=9$. Next, let $N_4=\{(x,y)\in G\times G\| x|=3,|y|=2\}$, then $x\in\{b^2,a^3b^2a,a^3b^3\}$. and $y\in\{a^2,b^3,a^2b^3\}$. Thus, $|N_4|=9$.

Case 3: Let $N_5=\{(x,y)\in G\times G\| x|=4,|y|=3\}$, then $x\in\{a,a^3,ab,ab^2,ab^3,b^2a,ba,a^2b^3, a^3b,a^3b^2,a^2b^2a,a^2ba\}$ and $y\in\{b^2,a^3b^2a,a^3b^3\}$. This implies $|N_5|=33$. Next, let $N_6=\{(x,y)\in$

$G\times G \| x| = 3, |y| = 4\}$, then $x \in \{b^2, a^3b^2a, a^3b^3\}$ and $y \in \{a, a^3, ab, ab^2, ab^3, b^2a, ba, a^2b^3, a^3b, a^3b^2, a^2b^2a, a^2ba\}$. Thus, $|N_6| = 33$.

Hence, $P_{copr}(\mathbb{Z}_2\times\mathbb{Z}_3\times\mathbb{Z}_4) = \dfrac{|N_1+N_2+N_3+N_4+N_5+N_6|}{|\mathbb{Z}_2\times\mathbb{Z}_3\times\mathbb{Z}_4|^2} = \dfrac{131}{576}$.

Let $G=\mathbb{Z}_3\times\mathbb{Z}_8$ where $G=\langle a,b \mid a^8=b^3=1, bab=a\rangle = \{e, a, a^2, a^3, a^4, a^5, a^6, a^7, b, a^7ba, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b, ba, aba, a^2ba, a^3ba, a^4ba, a^5ba, a^6ba\}$. It is found that $|e|=1$, $|a^4|=2$, $|a^2|=|b|=|a^7ba|=3$, $|a^6|=4$, $|a^4b|=|a^3ba|=6$, $|a|=|a^3|=|a^5|=|a^7|=|ab|=|a^3b|=|a^5b|=|a^7b|=|ba|=|a^2ba|=|a^4ba|=|a^6ba|=8$ and $|a^2b|=|a^6b|=|a^6b|=|a^5ba|=12$. Four cases should be considered as follows:

Case 1: If $x=e$, then x is co-prime to each element y in G since $(|x|,|y|)=(|e|,|y|)=1$. Let $N_1=\{(e,y)\in G\times G\}$. Then, $|N_1|=24$. Now, if $y=e$, then y is also co-prime to all x in $G\setminus e$ since $(|x|,|y|)=(|x|,|e|)=1$. Let $|N_2|=\{(x,e)\in G\setminus e\times G\}$. Then, $|N_2|=23$.

Case 2: Let $N_3=\{(x,y)\in G\times G \| x|=2, |y|=3\}$, then $x\in\{a^4\}$ and $y\in\{a^2, b, a^7ba\}$. This implies $|N_3|=3$. Next, let $N_4=\{(x,y)\in G\times G \| x|=3, |y|=2\}$, then $x\in\{a^2, b, a^7ba\}$ and $y\in\{a^4\}$. Thus, $|N_4|=3$.

Case 3: Let $N_5=\{(x,y)\in G\times G \| x|=4, |y|=3\}$, then $x\in\{a^6\}$ and $y\in\{a^2, b, a^7ba\}$. This implies $|N_5|=3$. Next, let $N_6=\{(x,y)\in G\times G \| x|=3, |y|=4\}$, then $x\in\{a^2, b, a^7ba\}$ and $y\in\{a^6\}$. Thus, $|N_6|=3$.

Case 4 : Let $N_7=\{(x,y)\in G\times G \| x|=3, |y|=8\}$, then $x\in\{a^2, b, a^7ba\}$ and $y\in\{a, a^3, a^5, a^7, ab, a^3b, a^5b, a^7b, ba, a^2ba, a^4ba, a^6ba\}$. This implies $|N_7|=36$. Next, let $N_8=((x,y)\in G\times G \| x|=3, |y|=4\}$, then $x\in\{a, a^3, a^5, a^7, ab, a^3b, a^5b, a^7b, ba, a^2ba, a^4ba, a^6ba\}$ and $y\in\{a^2, b, a^7ba\}$. Thus, $|N_8|=36$.

Hence, $P_{copr}(\mathbb{Z}_3\times\mathbb{Z}_8) = \dfrac{|N_1+N_2+N_3+N_4+N_5+N_6+N_7+N_8|}{|\mathbb{Z}_3\times\mathbb{Z}_8|^2} = \dfrac{131}{576}$.

Let $G=A_4\times\mathbb{Z}_2$ where $G=\langle a,b,c \mid a^2=b^3=c^2=1, ab=ba, ac=ca, c=bcbcb\rangle = \{\mathrm{e}, a, b, b^2, c, ab, ab^2, bc, b^2c, abc, ab^2c, ac, cb, acb, bcb, abcb, b^2cb, ab^2cb, cb^2, acb^2, bcb^2, abcb^2, b^2cb^2, ab^2cb^2\}$. It is found that $|e|=1$, $|a|=|c|=|ac|=|b^2cb|=|ab^2cb|=|bcb^2|=|abcb^2|=2$, $|b|=|b^2|=|bc|=|cb|=|b^2c|=|bcb|=|cb^2|=|b^2cb^2|=3$ and $|ab|=|ab^2|=|abc|=|ab^2c|=|acb|=|abcb|=|acb^2|=|ab^2cb^2|=6$. In order to find the $P_{copr}(G)$, two cases that should be considered are given as follows:

Case 1: If $x = e$, then x is co-prime to each element y in G since $(|x|,|y|) = (|e|,|y|) = 1$. Let $N_1 = \{(e, y) \in G \times G\}$. Then, $|N_1| = 24$. Now, if $y = e$, then y is also co-prime to all x in $G \setminus e$ since $(|x|,|y|) = (|x|,|e|) = 1$. Let $|N_2| = \{(x,e) \in G \setminus e \times G\}$. Then, $|N_2| = 23$.

Case 2: Let $N_3 = \{(x, y) \in G \times G \| x| = 2, |y| = 3\}$, then $x \in \{a, c, ac, b^2cb, ab^2cb, bcb^2, abcb^2\}$ and $y \in \{b, b^2, bc, b^2c, cb, bcb, cb^2, b^2cb^2\}$. This implies $|N_3| = 56$. Next, let $N_4 = \{(x, y) \in G \times G| |x| = 3, |y| = 2\}$, then $x \in \{b, b^2, bc, b^2c, cb, bcb, cb^2, b^2cb^2\}$. and $y \in \{a, c, ac, b^2cb, ab^2cb, bcb^2, abcb^2\}$. Thus, $|N_4| = 56$.

Hence, $P_{copr}(A_4 \times \mathbb{Z}_2) = \dfrac{|N_1 + N_2 + N_3 + N_4|}{|A_4 \times \mathbb{Z}_2|^2} = \dfrac{159}{576}$.

Let $G = D_{12}$ where $G = \langle a, b, c \mid a^{12} = b^2 = 1, bab = a^{-1} \rangle = \{e, a, a^2, a^3, a^4, a^5, a^6, a^7, a^8, a^9, a^{10}, a^{11}, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b, a^8b, a^9b, a^{10}b, a^{11}b\}$. It is found that $|e| = 1$, $|a^6| = |b| = |ab| = |a^2b| = |a^3b| = |a^4b| = |a^5b| = |a^6b| = |a^7b| = |a^8b| = |a^9b| = |a^{10}b| = |a^{11}b| = 2$, $|a^4| = |a^8| = 3$, $|a^3| = |a^9| = 4$, $|a^2| = |a^{10}| = 6$ and $|a| = |a^5| = |a^7| = |a^{11}| = 12$. Below are the cases that should be considered to find the $P_{copr}(G)$.

Case 1: If $x = e$, then x is co-prime to each element y in G since $(|x|,|y|) = (|e|,|y|) = 1$. Let $N_1 = \{(e, y) \in G \times G\}$. Then, $|N_1| = 24$. Now, if $y = e$, then y is also co-prime to all x in $G \setminus e$ since $(|x|,|y|) = (|x|,|e|) = 1$. Let $|N_2| = \{(x,e) \in G \setminus e \times G\}$. Then, $|N_2| = 23$.

Case 2: Let $N_3 = \{(x, y) \in G \times G \| x| = 2, |y| = 3\}$, then $x \in \{a^6, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b, a^8b, a^9b, a^{10}b, a^{11}b\}$ and $y \in \{a^4, a^8\}$. This implies $|N_3| = 26$. Next, let $N_4 = \{(x, y) \in G \times G \| x| = 3, |y| = 2\}$, then $x \in \{a^4, a^8\}$ and $y \in \{a^6, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b, a^8b, a^9b, a^{10}b, a^{11}b\}$. Thus, $|N_4| = 26$.

Hence, $P_{copr}(D_{12}) = \dfrac{|N_1 + N_2 + N_3 + N_4|}{|D_{12}|^2} = \dfrac{99}{576}$.

The Co-Prime Graph for Nonabelian Metabelian Groups of Order 24

In 2014, Ma et al. (2014) have extended the study of the prime graph to the co-prime graph. They determined the types of graph that can be possibly obtained together with some properties of the graph. Unfortunately, the focus of the paper is too general which is the finite group. Hence, an extensive study can be done as this research concentrates only on the nonabelian metabelian groups of order 24. In this section, D_{12} is taken as an example to explain the co-prime graph. From the graph obtained, the types of graph, the number of edges and some properties such as the dominating number and the independent number are calculated.

The co-prime graph of D_{12}

Let $D_{12}=\left\langle a,b \middle| a^{12}=b^2=1, bab=a^{-1}\right\rangle=\{e,a,a^2,a^3,a^4,a^5,a^6,a^7,a^8,a^9,a^{10},a^{11},b,ab,a^2b,a^3b,a^4b,a^5b,a^6b,$ $a^7b,a^8b,a^9b,a^{10}b,a^{11}b\}$. Therefore, the co-prime graph is illustrated below.

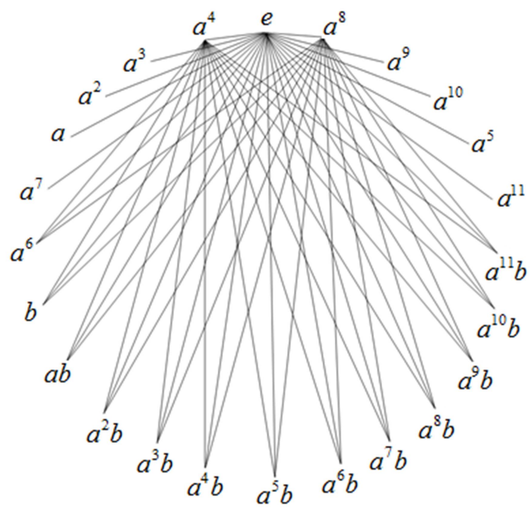

Figure 1: The Co-prime Graph of D_{12}

Hence, the total number of edges that connect to every vertex is 49. It can also be concluded that the graph is not a bipartite graph by using Proposition 1. Also, for the dominating number, $\gamma(\Gamma_{copr}(D_{12}))=1$ as it follows from Proposition 2. As for the independent number, from Definition 6, the possible independent sets are as follows;

Size 2 : $\{a,a^2\},\{a,a^3\},\{a^2,a^3\},\ldots$

Size 3 : $\{a,a^2,a^3\},\{a^2,a,a^7\},\{a^2,a,a^6\},\ldots$

Size 4 : $\{a,a^2,a^3,a^7\},\{a^2,a,a^7,a^6\},\{a,a^7,a^6,b\},\ldots$

⋮

Size 21 : $\{a,a^2,a^3,a^5,a^6,a^7,a^9,a^{10},a^{11},b,ab,a^2b,a^3b,a^4b,a^5b,a^6b,a^7b,a^8b,a^9b,a^{10}b,a^{11}b\}$.

Therefore, $\alpha(\Gamma)=21$ since the largest independent set is $\{a,a^2,a^3,a^5,a^6,a^7,a^9,a^{10},a^{11},b,ab,a^2b,$ $a^3b,a^4b,a^5b,a^6b,a^7b,a^8b,a^9b,a^{10}b,a^{11}b\}$. The same steps are repeated when $G=S_3\times\mathbb{Z}_2\times\mathbb{Z}_2$, $S_3\times\mathbb{Z}_4$, $(\mathbb{Z}_6\times\mathbb{Z}_2)\rtimes\mathbb{Z}_2$, $\mathbb{Z}_3\times Q$, $D_4\times\mathbb{Z}_3$, $Q\times\mathbb{Z}_3$, $\mathbb{Z}_2\times\mathbb{Z}_3\times\mathbb{Z}_4$, $\mathbb{Z}_3\rtimes\mathbb{Z}_8$ and $A_4\times\mathbb{Z}_2$ as G is nonabelian metabelian groups of order 24.

The complete results of the co-prime graph for nonabelian metabelian groups of order 24 can be referred to Table 1 below. The table below indicates the number of edges, the types of graph, the dominating number and the independent number which used the definitions, some propositions and theorem by Ma et al. (2014).

Table 1: The Co-prime Graph of Nonabelian Metabelian Groups of Order 24

No.	G	Number of Edges	Type of Graph	$\gamma(\Gamma)$	$\alpha(\Gamma)$
1	$S_3 \times \mathbb{Z}_4$	53	Not bipartite	1	21
2	$S_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	53	Not bipartite	1	21
3	$(\mathbb{Z}_6 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_2$	53	Not bipartite	1	21
4	$\mathbb{Z}_3 \times Q$	53	Not bipartite	1	21
5	$D_4 \times \mathbb{Z}_3$	37	Not bipartite	1	21
6	$Q \times \mathbb{Z}_3$	37	Not bipartite	1	21
7	$\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_4$	65	Not bipartite	1	20
8	$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	65	Not bipartite	1	20
9	$A_4 \times \mathbb{Z}_2$	79	Not bipartite	1	16
10	D_{12}	49	Not bipartite	1	21

CONCLUSION

Group and graph theory have its own importance. Group theory benefits more on the future research while graph theory have been used to solve problem such as the Konigsberg bridge problem and the hub and spoke model. This research focuses on both the group and graph theory whereby the co-prime probability and its related graphs for nonabelian metabelian groups of order 24 are determined. In the meantime, the number of edges, the types of graph, the dominating number and the independent number of the co-prime graph are also identified. It can be concluded that the co-prime probability together with the number of edges and the independent number for the graphs of the nonabelian metabelian groups of order 24 varies. The types of graph for the co-prime graph are not bipartite and the dominating number for all groups of nonabelian metabelian groups of order 24 are equal to one.

The co-prime probability and its related graphs can be extended by studying the nonabelian metabelian groups of order greater than 24. Other properties of the graphs such as the diameter, the girth, the chromatic number and the clique number can also be attained for the near future.

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