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Experience-Discovery Approach: The Effect of Open-Closed Method (OCM) in Teaching Chain Rule Differentiation of Composite Function

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ABSTRACT

This study applies an innovative technique called an Open-closed Method (OCM) used in teaching chain rule differentiation of composite function. The purpose of this study is to analyze the results of students after the implementation of OCM in class. 60 students from Diploma in Applied Sciences were chosen randomly to participate in this study and all of them have similar characteristics and intellectual ability. The students were divided into two groups; 30 students each for experimental and control group. Data were collected from students' test performance which covers a topic on differentiation and were analyzed using independent sample t-test to compare mean score between the two groups. Coefficient of variation was also used to identify the consistency of mean score in experimental group applying OCM compared to the mean score in control group that used the conventional technique. The findings of this study show that there is a significant difference in performance between these two groups where the experimental group performs much better than control group. The distribution of marks for experimental group is more consistent than control group. Therefore, the OCM is found to be a good technique in teaching chain rule differentiation and it helps students improve their performance in Calculus.

Keywords: Differentiation, Derivative, Calculus, Composite Function, Chain Rule Differentiation, Active Learning

INTRODUCTION

Calculus is a major branch in Mathematics and it also gives a great challenge to teaching Mathematics worldwide. The essence of Calculus is derivative which is used widely in differentiation and integration. Derivative is a way to find the rate of change of a function with respect to its variable while differentiation is a process of finding the derivative. There are several rules of differentiation that students should know which include constant rules, power rules, product rules, quotient rules, chain rules, trigonometric rules, logarithmic rules and exponential rules. Through experience in teaching Calculus, chain rule differentiation is one of the most challenging rules for students to comprehend because it involves a composite function. According to Gordon (2005), students have difficulties when it involves a composite function because they do not see where it comes from when the formula is expressed using symbols. Composition of function is also known as function in another function. All types of function can be put into another function including polynomials, trigonometry, logarithmic and exponential function. Horvath (2008) stated that teaching and learning chain rule is very complex even it seems quite simple looking at the notation. Most of students misunderstand between the composition of function and multiplication of functions. Findings by Asyura et al. (2017) identify that the highest percentage in mathematical misconception in course of Calculus 1 in UiTM Pahang are involving bracket and trigonometry properties which is mostly used in composite functions. When it comes to chain rule differentiation, students need to have a strong background in the topic of function. If they have a wrong concept in their mind, it will be a great challenge to undo the misconception. It is good to avoid the probability of misconception when introducing the topic to the students for the first time.

Students who take up calculus also need to have a strong foundation of Algebra. Somehow, when they learn new topics in Calculus, they tend to forget algebra. When teaching calculus, lecturers need to keep the algebra simple and spend more time and stress more on the concept of calculus. According to Ozken (2011), the main reason why students have problems in Mathematics is because they do not understand the basic concept at early stage. This statement is also supported by Cottrill (1999), who stresses on understanding composite function as the key factor of understanding chain rule. The absent of visual approach in current pedagogical practices may also explain the difficulty of many students when working with the chain rule. Students may have several rules in their mind but they do not recognize the relationship between the rules. The rules may include general power rule and chain rule itself. He believes that the chain rule itself is not difficult for students but the issue is the composition of function. The problem to be considered here is the topic before composite function where the students should have strong background knowledge on the topic of function. Previous researchers like Mathews (1989), proposed computer algebra system, muMATH which was developed in late seventies to verify chain rule and Thoo (1995), proposed the use of arrow or tree diagram as mnemonic devise to explain chain rule. Tall (1993), has an opinion that visual alone is not enough; he suggests visual should be combined with great ability in numeric and symbolic. In fact, Millan (2001), integrates graphic, numeric and symbolic in his work with mathematical software namely Mathematica. In addition, Adem et al. (2010) had discussed about teaching and learning Mathematics using software. The computational commands from Mathematics software such as MAPLE can help the student to study the techniques of computation while solving the problems. As an example for differentiation, MAPLE will specify the way of differentiation rule that can be used at each step. The interactive commands will help Students to explore the concepts of differentiation rule. Two common mistakes done by teachers when teaching functions which are poor definition of function and poor notation as well as the way it is presented to students. Teachers should avoid showing the definition and letting the students figure it out by themselves because they may get it wrong (Bayens, 2016). Bayens (2016) suggested an alternative method of teaching function called as an active learning and he placed it into four categories, namely visual, kinesthetic, auditory, technological, or whole class involvement. Ordinary style of lecture by showing the definition and formula only is not enough for teaching and learning calculus. Lecture should be supported by active learning using interactive and creative teaching method for the creation of an environment conducive for learning and hence, it is not only improves students' understanding but also improve students' performance in calculus (Tall, 1993; Bayens, 2016).

Due to the challenges in teaching chain rule differentiation of composite function, we explore a technique namely as Open-closed Method (OCM). OCM consist of an interesting visual, numeric and symbolic aid that can be written on whiteboard or computerized and also can be converted in the video form. OCM used to help students in understanding the differentiation rule involves in differentiating the composite function as it provides visual method that guided students to find the solution step by step. Mainly, the aim of this study is to analyze the result of students after the implementation of OCM in teaching chain rule differentiation in class.

OPEN CLOSED METHOD (OCM)

OCM is an innovative and interactive teaching technique used in teaching differentiation of composite functions. In general, this technique follows the chain rule differentiation and the generalized rule differentiation. The OCM starts by differentiating the outer function and continue on closing the function that has been differentiated then differentiate the remaining function. The step is continued and will end until the function is finally closed and solved. It is a very effective technique which guides students to use formula correctly.

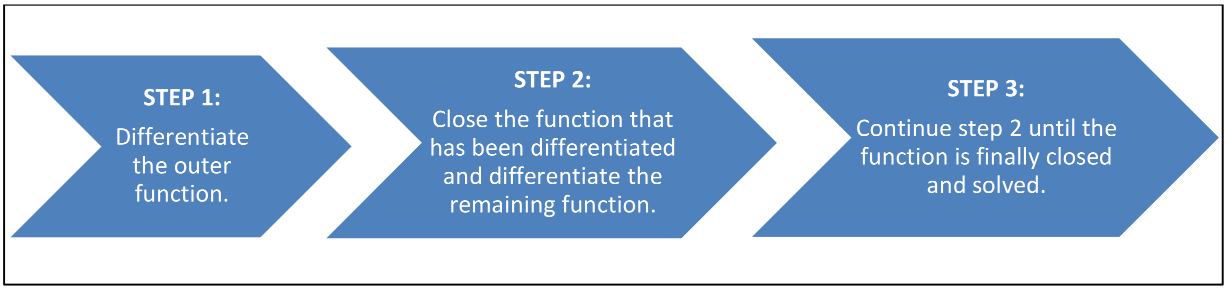

FIGURE 1: General step of Open-close Method (OCM)

The Generalize Power Rule Differentiation for composite function $y = f\left[g(x)\right]$ is given as below:

$$y' = f'\left[g(x)\right] \cdot g'(x) \tag{1}$$

Thus, OCM gives a general idea of differentiating the composite function by using the following formula:

$$y' = \left(\text{Differentiating the outer function}\right) \times \left(\text{Differentiating the inner function}\right) \tag{2}$$

The following example shows the step on how to find the first derivative of $f(x) = \sqrt{2x-3}$ using formula and using OCM. By using formula of generalize power rule,

$$\text{If} \quad f(x) = (ax+b)^n \quad \text{then} \quad f'(x) = n(ax+b)^{n-1}\frac{d}{dx}(ax+b) \tag{3}$$

Thus, the first derivative of $f(x) = \sqrt{2x-3}$ is given by

$$f'(x) = \frac{1}{2}(2x-3)^{\frac{1}{2}-1}\frac{d}{dx}(2x-3) = \frac{1}{2}(2x-3)^{-\frac{1}{2}}(2) = (2x-3)^{-\frac{1}{2}} \tag{4}$$

Basically, this is a simple example to show an overview on how to use OCM and figure 2 shows the steps to find the first derivative of $f(x) = \sqrt{2x-3}$ using OCM.

Next, the following example shows the step on how to find the first derivative of a composite function that composes trigonometric function using formula and steps using OCM.

Given,

$$f(x) = \sin^3\left(2x^2-3\right)^4 = \left[\sin\left(2x^2-3\right)^4\right]^3 \tag{5}$$

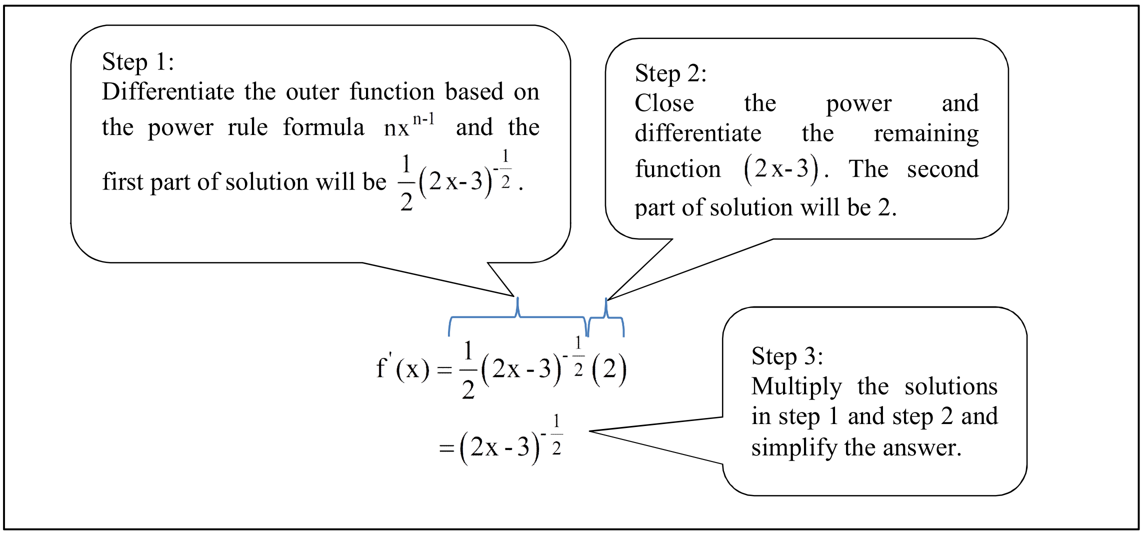

FIGURE 2: Steps to find the first derivative of $f(x) = \sqrt{2x-3}$ using OCM.

By using formula, the first derivative of $f(x) = \sin^3 (2x^2-3)^4$ is given as follow:

$$\begin{aligned} f'(x) &= 3\left[\sin\left(2x^2-3\right)^4\right]^2 . \frac{d}{dx}\left[\sin\left(2x^2-3\right)^4\right] \\ &= 3\left[\sin\left(2x^2-3\right)^4\right]^2 .\cos\left(2x^2-3\right)^4 . \frac{d}{dx}\left[\left(2x^2-3\right)^4\right] \\ &= 3\left[\sin\left(2x^2-3\right)^4\right]^2 .\cos\left(2x^2-3\right)^4 .4\left(2x^2-3\right)^3 \frac{d}{dx}\left(2x^2-3\right) \qquad (6) \\ &= 3\left[\sin\left(2x^2-3\right)^4\right]^2 .\cos\left(2x^2-3\right)^4 .4\left(2x^2-3\right)^3 .4x \\ &= 48\left(2x^2-3\right)^3 \sin^2\left(2x^2-3\right)^4 .\cos\left(2x^2-3\right)^4 \end{aligned}$$

In Figure 3 below, it shows the explanations on how to find the first derivative of $f(x) = \sin^3 (2x^2-3)^4$ using OCM. Deep understanding from students' point of view can be gained if they are asked to explain the differentiation rule involved in each step. Lecturers can focus more on asking students to solve the most relevant problems to the topic discussed in class so that student can explore and understand the concept of chain rule differentiation deeply.

METHODOLOGY

This preliminary study is conducted on sixty students of Diploma in Applied Sciences in UiTM Pahang that were chosen randomly. The students were divided into two groups; 30 students each for experimental and control group. Open-closed Method (OCM) as an alternative technique of teaching chain rule differentiation were applied on the experimental group to enhance students' understanding on chain rule differentiation of composite function. Due to time constrain to finish the syllabus given in one semester; only a group of 30 students were experimented by introducing this method. OCM is used by the experimental group throughout the learning process on topic differentiation in three weeks. Then, they were given a test that consists of this topic for assessment. The test consists of 10 questions with a total of 40 marks for specific topic on differentiation. Their scores were compared to the control group of 30 students that does not implement OCM which given the same set of questions. The test was graded by their lecturers and each student was informed of their score. Scores for the question involving chain rule differentiation were extracted from experimental and control group. Data were collected from students' test performance which covers the topic on differentiation. The scores collected were analyzed using statistical procedures executed by SPSS which included descriptive statistics, independent t-test and coefficient of variation.

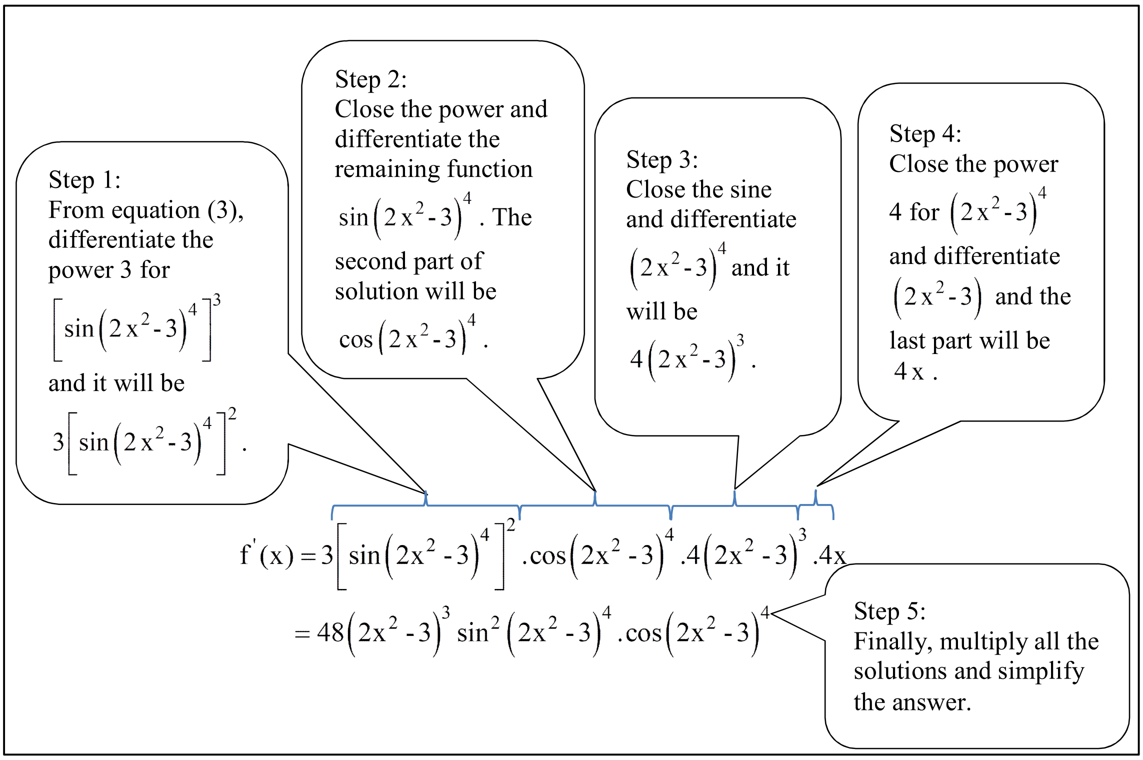

FIGURE 3: Steps to find the first derivative of $f(x) = \sin^3\left(2x^2-3\right)^4$ using OCM.

RESULT

The table below shows the mean and standard deviation of the scores for all students. All data gathered were normally distributed using the value of skewness ±1. An independent t-test was conducted to determine whether there was a significant difference in the mean score between students in experimental group and control group. Differences were considered significant at p-value < 0.05. The coefficient of variations was then measured to determine which group of students had more consistent distribution of mean score. The smaller value indicates the more consistent distribution.

TABLE 1. Independent sample t-test and coefficient of variation analysis

Marks	M(SD)	95% Confidence Interval of The Difference	t-test(p-value)	Coefficient of variation (%)
EG (n=30)	27.88(2.55)	(4.67, 10.75)	5.17(0.00)*	9.15%
CG (n=30)	20.17(7.33)			36.34%

M=mean, SD=standard deviation, *p-value<0.05

According to TABLE 1, the average score by experimental group students is 27.88 while the average score by control group students is 20.17. Standard deviation for experimental group is 2.55 which shows the distribution of score smaller than standard deviation for control group (7.33). Students that used OCM scored on average of 7.71 marks higher than students who learned using conventional method. There is evidence that there is a significant difference between the mean test score of the experimental group and the control group. The test shows that the difference scores are statistically significant lies between 4.67 and 10.75 with statistically significant t-value = 5.17, $p < 0.05$. This result is supported by the value of coefficient of variation for experimental group (9.15%) which is smaller than control group. The distribution of scores for experimental group is more consistent than control group. Therefore, it can be concluded that students who were exposed the use of OCM in differentiation of composite function achieved better results in the test for that specific topic than the control group.

DISCUSSION AND CONCLUSION

There are many ways to make teaching mathematics better. Base on this study, it is respectable to involve students with active learning where the students should be engage to do the open and close process using OCM. Prior to the OCM, there must be a clear definition of composite function as mentioned by Bayens (2016). This can be perceived as an alternative method to use in teaching chain rule beside the use of arrow and tree diagram (Thoo, 1995) and muMATH (Mathews, 1989). This study provides an innovative and interactive visual method using OCM as supporting material to be used efficiently for more understanding about chain rule. When teachers introduce a new topic to students, the definitions of important terms must be well explained to avoid misconceptions. As mentioned by Horvath (2008), students might have misunderstood by looking at the notation eventhough most of them gave the correct answer. This method helps to reduce the initial tendency that students treating the composite function as the multiplication of function.

There are many studies that reported on the students' difficulties in learning calculus. On that reason, teachers and educators should explore and find alternative way in teaching calculus (Gordon, 2005; Horvath, 2008; Park and Lee, 2016). Study from Park and Lee (2016) confirmed that the use of abduction in mathematical modelling helps students in generalization of chain rule's model. This paper presented a new idea and a creative way to teach the topic that students often dislike. The result of this study shows that OCM gives a positive impact towards the students' result on chain rule differentiation which students who were exposed the use of OCM achieved better results. The result was supported by Andrade-Aréchiga et al. (2012) that experimental group using interactive platform for learning calculus has positive effect toward motivation in learning calculus and academic performance of student. The limitation of this study is; it only describes this set of students rather than the entire population of first year calculus students. It is recommended to use this method to the entire population of students who learn Calculus. In the future studies, OCM can be applies in differentiating implicit functions and functions with more than one variable. In the long run, this paper encourages current and future teachers to use an alternative method in teaching chain rule differentiation of composite function to facilitate students understand the key ideas of calculus.

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REFERENCES

Adem, K., Munther A. H. and Said Husainc, S.K. (2010). Teaching and Learning using Mathematics Software ”The New Challenge”. *Procedia Social and Behavioral Sciences* 8, 613–619.

Andrade-Aréchiga, M., López, G. and López-Morteo, G. (2012). Assessing effectiveness of learning units under the teaching unit model in an undergraduate mathematics course. *J. Computer & Education*, 59, 594-606.

Asyura, A. N., Abdullah, N. H. M., Ahmad, S., Nor Habibah, T. and Idris, A. S. (2017). Mathematical misconception in calculus 1: Identification and gender difference. Proceedings of the 24th National Symposium on Mathematical Sciences, *AIP Conf. Proc.* 1870, 070001-1–070001-7.

Bayens, M. (2016). *Teaching Functions: The Good, the Bad, and the Many Ways to Do Better*. Honors College thesis, Murray States University.

Cottrill, J. F. (1999). *Students' Understanding of the Concept of Chain Rule in First Year Calculus and the Relation to their Understanding of Composition of Functions*. PhD thesis, Purdue University.

Gordon, S. P. (2005). Discovering the Chain Rule Graphically. *J. Maths. and Computer Education*. 39(3), 195-197.

Horvath, A. (2008). *Looking at Calculus Students' Understanding from the inside-out: The Relationship Between the Chain Rule and Function Composition*. CRUME 2008. Electronic Proceeding, http://sigmaa.maa.org/rume/crume2008.

Mathews, J. H. (1989). Teaching Differentiation with muMATH. *Computers in Mathematics and Science Teaching, Spring 1989*, 8(3). 50-53.

Millan, J. R. (2001). Integrated Symbolic-Graphic-Numeric Analysis and Design in Nonlinear Control through Notebooks in Mathematica. *J. (eds) Comp Aided Syst Theory*, EUROCAST 2001, 2178, 405-420.

Ozken, E. M. (2011). Misconceptions in Radicals in High School Mathematics. *J. Soc. and Beh. Sc.* 15, 120-127.

Park, J. H. and Lee, K. -H. (2016). How can Students Generalize the Chain Rule? The Roles of Abduction in Mathematical Modeling. *Eurasia J. Mathematics, Science & Technology Education*, 12(9), 2331-2352.

Tall, D. (1993). Students' Difficulties in Calculus. ICME-7 1992, 13-28.

Thoo, J. B. (1995). Composition and the Chain Rule using Arrow Diagrams. PRIMUS, 5, 291–295.