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Using Predictor-Corrector Methods For Solving Fuzzy Differential Equations

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ABSTRACT

In this paper we studied the Fuzzy Differential Equations and we used Predictor-Corrector methods for solving these equations (these methods are Multi-step methods), we used two methods Adams-Bashforth-Moulton three-step method and Adams-Bashforth-Moulton four-step method, and we compared the solution of each of method with the exact solution, and then compared the solution of each of them with the others.

Keywords: Fuzzy Differential equations, Multi-step methods, Predictor-Corrector methods, 3-step Adams-Bashforth-Moulton method, 4-step Adams-Bashforth-Moulton method.

INTRODUCTION

The Fuzzy Differential Equations (FDEs) are utilized for the purpose of the modeling problems in Science and Engineering. Most of the problems in Science and Engineering require the solution of a Fuzzy Differential Equation (FDE) which are satisfied in Fuzzy initial conditions, therefore, a fuzzy initial value problem is occurs and should be solved. It is too complicated to obtain the exact solution of (FDE) which models the mentioned problems.

The concept of a fuzzy derivative was first introduced by Chang and Zedeh (19720; it was followed by Dubois and Prade (1982), who defined and used the extension principle. Other methods have been discussed by Puri and Ralescu (1983), Goetschel and Voxman (1986), and Kandel (1980) and Kandel and Byatt (1978) who applied the concept of FDEs to the analysis of fuzzy dynamical problems. The FDE and the fuzzy initial value problem (Cauchy problem) were regularly treated by Kaleva (1987,1990), Seikkala (1988), He and Yi (1989), Kloeden (1991), Menda (1988), Nieto and Rodriguez-Lopez (2006) and by many other researchers (see Bede and Gal (2005), Bede et al. (2006), Buckley and Feuring (1999), Jowers et al. (2007). The numerical methods for solving FDEs are introduced in Abbasbandy and Allahviranloo (2002), Abbasbandy et al. (2004), Allahviranloo et al.(2007) and Ma et al. (1999).

In the present note it is shown that it is possible to translate a FDE into a system of ODEs, For this aim, as a generalization of some results in Kaleva (2006), Chalco-Cano and Roman-Flores (2006), Stefanini(2007) and Bede at al. (2007), we present characterization theorems for the solution of a FDE under the Hukuhara derivative-based interpretation, by the solution of a system of ODEs. So, in order to obtain numerical solutions of Fuzzy Differential Equations under Hukuhara differentiability, it is not necessary to rewrite on numerical solutions of ODEs in the fuzzy setting, but instead we can use any numerical method for the ODEs directly.

In Section 2, Some basic definitions and theorems are brought, Adams-Bashforth-Moulton three-step method for solving Fuzzy Differential Equations are introduced in Section 3, In Section 4, Adams-Bashforth-Moulton four-step method for solving Fuzzy Differential Equations are proposed. Two examples are presented in Section 5, and finally the Conclusion is drawn in Section 6.

DEFINITIONS AND THEOREMS

Definition 2.1. An m-step method for solving the initial-value problem is one whose difference equation for finding the approximation $y(t_{i+1})$ at the mesh point t_{i+1} can be represented by the following equation: (Allahviranloo et al., 2007)

$$y(t_{i+1}) = a_{m-1}y(t_i) + a_{m-2}y(t_{i-1}) + ... + a_0 y(t_{i+1-m}) + h\{b_m f(t_{i+1}, y_{i+1}) + b_{m-1} f(t_i, y_i) + ...$$
$$+ b_0 f(t_{i+1-m}, y_{i+1-m})\} \quad (2.1)$$

For $i = m-1, m, ..., N-1$, such that $a = t_0 \leq t_1 \leq .... \leq t_N = b,\ h = \frac{(b-a)}{N} = t_{i+1} - t_i$ and $a_0, a_1, ..., a_{m-1}, b_0, b_1, ..., b_m$ are constants with the starting values:

$$y_0 = \alpha_0, y_1 = \alpha_1, y_2 = \alpha_2, ..., y_{m-1} = \alpha_{m-1}.$$

When $b_m = 0$, the method is known as explicit, Since Eq. (2.1) gives y_{i+1} explicit in terms of previously determined values. When $b_m \neq 0$, the method is known as implicit, since y_{i+1} occurs both sides of Eq. (2.1) and is specified only implicitly.

Definition 2.2. Associated with the difference equation: (Allahviranloo et al., 2007)

$$y_{i+1} = a_{m-1}y_i + a_{m-2}y_{i-1} + ... + a_0 y_{i+1-m} + hf(t_i, h, y_{i+1}, y_i ......, y_{i+1-m}),$$
$$y_0 = \alpha_0, y_1 = \alpha_1, y_2 = \alpha_2, ..., y_{m-1} = \alpha_{m-1}. \quad (2.2)$$

The following, called the characteristic polynomial of the method is:

$$p(\lambda) = \lambda^m - a_{m-1}\lambda^{m-1} - a_{m-2}\lambda^{m-2} - .... - a_1\lambda - a_0.$$

If $|\lambda_i| \leq 1$ for each $i = 1, 2, ..., m,$ and all roots with absolute value 1 are simple roots, then the difference method is said to satisfy the root condition.

Theorem 2.1. A multi-step method of the form (2.2) is stable if and only if it satisfies the root condition. (A tilde is placed over a symbol to denote a fuzzy set so $\tilde{\alpha}_1, \tilde{f}(t), ...$) (Allahviranloo et al., 2007).

Definition 2.3. An arbitrary fuzzy number with an ordered pair of functions $(\underline{u}(\alpha), \overline{u}(\alpha)),\ 0 \leq \alpha \leq 1$, which satisfy the following requirements is represented: (Allahviranloo et al., 2007)

- $\underline{u}(\alpha)$ is a bounded left continuous nondecreasing function over [0,1].
- $\overline{u}(\alpha)$ is a bounded left continuous nonincreasing function over [0,1].
- $\underline{u}(\alpha) \leq \overline{u}(\alpha),\ 0 \leq \alpha \leq 1.$

Definition 2.4. Let I be a real interval. A mapping $y: I \rightarrow E$ is called a fuzzy process and its α-level set is denoted by: (Allahviranloo et al., 2007)

$$[y(t)]^\alpha = [\underline{y}^\alpha(t), \overline{y}^\alpha(t)], \quad t \in I, \quad \alpha \in (0,1]. \quad (2.3)$$

Definition 2.5. Let R_F denote the class of fuzzy numbers and let $x, y \in R_F$. If there exists $z \in R_F$ such that $x = y + z,$ then z is called the Hukuhara difference of x and y and it is denoted by $x(-)y$. (Kaleva, 2006)

Note: the "$(-)$" sign stands always for Hukuhara difference and $x(-)y \neq x + (-1)y.$

Definition 2.6. Let $f:(a,b)\to R_F$ and $x_0\in(a,b)$. We say that *f* is Hukuhara differentiable at x_0, if there exists an element $f'(x_0)\in R_F$, such that for all $h>0$ sufficiently small, $\exists f(x_0+h)(-)f(x_0),\ f(x_0)(-)f(x_0-h)$ and the limits: (Puri,1983)

$$\lim_{h\to 0}\frac{f(x_0+h)(-)f(x_0)}{h}=\lim_{h\to 0}\frac{f(x_0)(-)f(x_0-h)}{h}=f'(x_0) \tag{2.4}$$

Theorem 2.2. Let $F:(a,b)\to R_F$ be Hukuhara differentiable and denote $[F(t)]^\alpha=[\underline{F}^\alpha(t),\overline{F}^\alpha(t)]$. Then the boundary functions $\underline{F}^\alpha(t)$ and $\overline{F}^\alpha(t)$ are differentiable and: (Kaleva, 2006)

$$[F'(t)]^\alpha=[(\underline{F}^\alpha)'(t),(\overline{F}^\alpha)'(t)],\quad \alpha\in[0,1]. \tag{2.5}$$

Definition 2.7. Let us consider the fuzzy initial value problem (FIVP): (Kaleva, 2006)

$$\begin{cases} y'=f(t,y)\\ y(t_0)=y_0 \end{cases} \tag{2.6}$$

where $f:[t_0,t_0+a]\times R_F\to R_F$ and $y_0\in R_F$. Then the above theorem shows us a way how to translate the FIVP (2.6) into a system of ODEs.

Definition 2.8. Let $[y(t)]^\alpha=[\underline{y}^\alpha(t),\overline{y}^\alpha(t)]$. If *y(t)* is Hukuhara differentiable then $[y'(t)]^\alpha=[(\underline{y}^\alpha)'(t),(\overline{y}^\alpha)'(t)]$. So Eq. (2.6) translates into the following systems of ODEs: (see Kaleva(2006))

$$\begin{cases} (\underline{y}^\alpha(t))'=\underline{f}^\alpha(t,\underline{y}^\alpha(t),\overline{y}^\alpha(t))\\ (\overline{y}^\alpha(t))'=\overline{f}^\alpha(t,\underline{y}^\alpha(t),\overline{y}^\alpha(t))\\ \underline{y}^\alpha(t_0)=\underline{y}_0{}^\alpha\\ \overline{y}^\alpha(t_0)=\overline{y}_0{}^\alpha \end{cases} \tag{2.7}$$

where:

$$[f(t,y)]^\alpha=[\underline{f}^\alpha(t,\underline{y}^\alpha,\overline{y}^\alpha),\ \overline{f}^\alpha(t,\underline{y}^\alpha,\overline{y}^\alpha)]$$

Definition 2.9. The Seikkala derivative $y'(t)$ of a fuzzy process *y* (defined by Eq. (2.3)) is defined by: (Allahviranloo et al., 2007)

$$[y'(t)]^\alpha=[(\underline{y}^\alpha)'(t),(\overline{y}^\alpha)'(t)],\quad 0<\alpha\le 1. \tag{2.8}$$

Remark 2.1. If $y:I\to E$ is Seikkala differentiable and its Seikkala derivative *y*' is integrable over [0,1], then: (Allahviranloo et al., 2007)

$$y(t)=y(t_0)+\int_{t_0}^{t}y'(s)ds. \tag{2.9}$$

For all values of t_0,t where $t_0,t\in I$.

ADAMS-BASHFORTH-MOULTON THREE-STEP METHOD

The Adams-Bashforth-Moulton predictor-corrector method is a multi-step method derived from the fundamental theorem of calculus:

$$y(t_{i+1})=y(t_i)+\int_{t_i}^{t_{i+1}}f(t,y(t))dt \tag{3.1}$$

We are going to solve fuzzy initial value problem $y'(t) = f(t, y(t))$ by Adams-Bashforth-Moulton Three-Step Method. Let the fuzzy initial values be $\tilde{y}(t_i), \tilde{y}(t_{i-1}), \tilde{y}(t_{i-2})$, i.e.: (Allahviranloo et al., 2007)

$$\tilde{f}(t_i, y(t_i)), \quad \tilde{f}(t_{i-1}, y(t_{i-1})), \quad \tilde{f}(t_{i-2}, y(t_{i-2})).$$

The predictor used the Lagrange polynomial approximation for *f(t,y(t))*. It is integrated over the interval $[t_i, t_{i+1}]$ in (3.1). This process produced the Adams-Bashforth three-step method, Therefore, the Adams-Bashforth three-step method is obtained as follows: (Allahviranloo et al., 2007)

$$\begin{cases} \underline{y}^{\alpha}(t_{i+1}) = \underline{y}^{\alpha}(t_i) + \dfrac{h}{12}\left[5\underline{f}^{\alpha}(t_{i-2}, y(t_{i-2})) - 16\underline{f}^{\alpha}(t_{i-1}, y(t_{i-1})) + 23\underline{f}^{\alpha}(t_i, y(t_i))\right] \\ \bar{y}^{\alpha}(t_{i+1}) = \bar{y}^{\alpha}(t_i) + \dfrac{h}{12}\left[5\bar{f}^{\alpha}(t_{i-2}, y(t_{i-2})) - 16\bar{f}^{\alpha}(t_{i-1}, y(t_{i-1})) + 23\bar{f}^{\alpha}(t_i, y(t_i))\right] \end{cases} \quad (3.2)$$

The corrector is developed similarly. The values $\underline{y}^{\alpha}(t_{i+1}), \bar{y}^{\alpha}(t_{i+1})$ just computed in (3.2) can be used in the Adams-Moulton method, So, the fuzzy initial value problem $y'(t) = f(t, y(t))$ can be solved by Adams-Moulton three-step method as follows: (Allahviranloo et al., 2007)

$$\begin{cases} \underline{y}^{\alpha}(t_{i+1}) = \underline{y}^{\alpha}(t_i) + \dfrac{h}{12}\left[-\underline{f}^{\alpha}(t_{i-2}, y(t_{i-2})) + 8\underline{f}^{\alpha}(t_{i-1}, y(t_{i-1})) + 5\underline{f}^{\alpha}(t_i, y(t_i))\right] \\ \bar{y}^{\alpha}(t_{i+1}) = \bar{y}^{\alpha}(t_i) + \dfrac{h}{12}\left[-\bar{f}^{\alpha}(t_{i-2}, y(t_{i-2})) + 8\bar{f}^{\alpha}(t_{i-1}, y(t_{i-1})) + 5\bar{f}^{\alpha}(t_i, y(t_i))\right] \end{cases} \quad (3.3)$$

ADAMS-BASHFORTH-MOULTON FOUR-STEP METHOD

The Adams-Bashforth-Moulton four-step method is also derived from (3.1), It is also used Lagrange polynomial approximation for *f(t,y(t))*, Now we are going to solve fuzzy initial value problem $y'(t) = f(t, y(t))$ by Adams-Bashforth four-step method (predictor method). Let the fuzzy initial values be $\tilde{y}(t_i), \tilde{y}(t_{i-1}), \tilde{y}(t_{i-2}), \tilde{y}(t_{i-3})$, i.e.: (Allahviranloo et al., 2007)

$$\tilde{f}(t_i, y(t_i)), \quad \tilde{f}(t_{i-1}, y(t_{i-1})), \quad \tilde{f}(t_{i-2}, y(t_{i-2})), \quad \tilde{f}(t_{i-3}, y(t_{i-3}))$$

Therefore, Adams-Bashforth four-step method is obtained as follows: [3]

$$\begin{cases} \underline{y}^{\alpha}(t_{i+1}) = \underline{y}^{\alpha}(t_i) + \dfrac{h}{24}\left[-9\underline{f}^{\alpha}(t_{i-3}, y(t_{i-3})) + 37\underline{f}^{\alpha}(t_{i-2}, y(t_{i-2})) - 59\underline{f}^{\alpha}(t_{i-1}, y(t_{i-1})) + 55\underline{f}^{\alpha}(t_i, y(t_i))\right] \\ \bar{y}^{\alpha}(t_{i+1}) = \bar{y}^{\alpha}(t_i) + \dfrac{h}{24}\left[-9\bar{f}^{\alpha}(t_{i-3}, y(t_{i-3})) + 37\bar{f}^{\alpha}(t_{i-2}, y(t_{i-2})) - 59\bar{f}^{\alpha}(t_{i-1}, y(t_{i-1})) + 55\bar{f}^{\alpha}(t_i, y(t_i))\right] \end{cases} \quad (4.1)$$

With similar way the fuzzy initial value problem $y'(t) = f(t, y(t))$ can be solved by Adams-Moulton four-step method as follows: (Allahviranloo et al., 2007)

$$\begin{cases} \underline{y}^{\alpha}(t_{i+1}) = \underline{y}^{\alpha}(t_i) + \dfrac{h}{24}\left[\underline{f}^{\alpha}(t_{i-3}, y(t_{i-3})) - 5\underline{f}^{\alpha}(t_{i-2}, y(t_{i-2})) + 19\underline{f}^{\alpha}(t_{i-1}, y(t_{i-1})) + 9\underline{f}^{\alpha}(t_i, y(t_i))\right] \\ \bar{y}^{\alpha}(t_{i+1}) = \bar{y}^{\alpha}(t_i) + \dfrac{h}{24}\left[\bar{f}^{\alpha}(t_{i-3}, y(t_{i-3})) - 5\bar{f}^{\alpha}(t_{i-2}, y(t_{i-2})) + 19\bar{f}^{\alpha}(t_{i-1}, y(t_{i-1})) + 9\bar{f}^{\alpha}(t_i, y(t_i))\right] \end{cases} \quad (4.2)$$

EXAMPLES

Example 5.1. Consider the fuzzy initial value problem:

$$\tilde{y}'(t) = -\tilde{y}(t) + t + 1$$
$$\tilde{y}(0) = (0.96, 1.01) \qquad (5.1.1)$$

As we know the exact solution of the Differential equation (5.1.1) is:

$$y(t) = t + (e^{-t})c_1 \qquad (5.1.2)$$

So, the exact solution by using the fuzzy initial condition is:

$$y(t) = t + (0.96, 1.01)e^{-t} \qquad (5.1.3)$$

Which is obtained by solving formally the system and using fuzzy numbers in the classical solution. By using the results of Definition (2.8) the solution is a Double number valued function and the endpoints are solutions of the system:

$$\underline{y}'(t) = -\underline{y}(t) + t + 1$$
$$\bar{y}'(t) = -\bar{y}(t) + t + 1$$
$$\underline{y}(0) = 0.96$$
$$\bar{y}(0) = 1.01 \qquad (5.1.4)$$

Having solution:

$$\underline{y}(t) = t - 0.025e^{t} + 0.985e^{-t}$$
$$\bar{y}(t) = t + 0.025e^{t} + 0.985e^{-t} \qquad (5.1.5)$$

By using Adams-Bashforth-Moulton three-step method with *N=10*, the following results are obtained:

Table (5.1.1): Numerical solution of Example (5.1) by Adams-Bashforth-Moulton three-step method

T	$\underline{y}(t)$	$\underline{y}(t)$ *exact*	*Error*	$\bar{y}(t)$	$\bar{y}(t)$ *exact*	*Error*
0	0.96	0.96	0	1.01	1.01	0
0.1	0.96860000	0.96864392	4.3921e-005	1.01390000	1.01388579	1.4208e-005
0.2	0.98590000	0.98598152	8.1523e-005	1.02690000	1.02691806	1.8061e-005
0.3	1.01110000	1.01118549	8.5492e-005	1.04820000	1.04822640	2.6403e-005
0.4	1.04343413	1.04340724	7.3112e-005	1.0770037	1.0770232	1.9462e-005
0.5	1.08220699	1.08226943	6.2442e-005	1.11258225	1.11259596	1.3707e-005
0.6	1.12680600	1.12685917	5.3168e-005	1.15429085	1.15429975	8.8970e-006
0.7	1.17667679	1.17672189	4.5093e-005	1.20154627	1.20155115	4.8779e-006
0.8	1.23131773	1.23135580	3.8074e-005	1.25382071	1.25382225	1.5430e-006
0.9	1.29027489	1.29030687	3.1982e-005	1.31063665	1.31063535	1.2013e-006
1	1.35313755	1.35316426	2.6704e-005	1.37156167	1.3715582	3.4373e-006

And by using Adams-Bashforth-Moulton four-step method with *N=10*, the following results are obtained:

Table (5.1.2): Numerical solution of Example (5.1) by Adams-Bashforth-Moulton four-step method

T	$\underline{y}(t)$	$\underline{y}(t)$ *exact*	*Error*	$\bar{y}(t)$	$\bar{y}(t)$ *exact*	*Error*
0	0.96	0.96	0	1.01	1.01	0
0.1	0.96860000	0.96864392	4.3921e-005	1.01390000	1.01388579	1.4208e-005
0.2	0.98590000	0.98608152	8.1523e-005	1.02690000	1.02691806	1.8061e-005
0.3	1.01110000	1.01128549	8.5492e-005	1.04820000	1.04822640	2.6403e-005
0.4	1.04342967	1.04350724	7.7567e-005	1.0769991	1.0770232	2.4049e-005
0.5	1.08228082	1.08236943	7.7567e-005	1.11257387	1.11259596	2.2088e-005
0.6	1.12686923	1.12695917	6.4062e-005	1.15427949	1.15429975	2.0253e-005
0.7	1.17673078	1.17672189	5.8183e-005	1.20153260	1.20155115	1.8554e-005
0.8	1.23136365	1.23145580	5.2843e-005	1.25380525	1.25382225	1.6995e-005
0.9	1.29031379	1.29030687	4.7992e-005	1.31061979	1.31063535	1.5565e-005
1	1.35317036	1.35326426	4.3586e-005	1.37154398	1.3715582	1.4253e-005

Now we compare the results of Table (5.1.1) by using Adams-Bashforth-Moulton three-step method with the results of Table (5.1.2) by using Adams-Bashforth-Moulton four-step method, the results with *N=10* are obtained:

Table (5.1.3): Comparison the results of Adams-Bashforth-Moulton three-step method with the results of Adams-Bashforth-Moulton four-step method

t	$\underline{y}(t)3-step$	$\underline{y}(t)4-step$	*Error*	$\bar{y}(t)3-step$	$\bar{y}(t)4-step$	*Error*
0	0.96	0.96	0	1.01	1.01	0
0.1	0.96860000	0.96860000	0	1.01390000	1.01390000	0
0.2	0.98590000	0.98590000	0	1.02690000	1.02690000	0
0.3	1.01110000	1.01110000	0	1.04820000	1.04820000	0
0.4	1.04343413	1.04342967	4.4549e-006	1.0770037	1.0769991	4.5868e-006
0.5	1.08220699	1.08228082	8.0739e-006	1.11258225	1.11257387	8.3808e-006
0.6	1.12680600	1.12686923	1.0894e-005	1.15429085	1.15427949	1.1356e-005
0.7	1.17667679	1.17673078	1.3090e-005	1.20154627	1.20153260	1.3676e-005
0.8	1.23131773	1.23136365	1.4769e-005	1.25382071	1.25380525	1.5452e-005
0.9	1.29027489	1.29031379	1.6010e-005	1.31063665	1.31061979	1.6766e-005
1	1.35313755	1.35317036	1.6881e-005	1.37156167	1.37154398	1.7690e-005

Example 5.2. Consider the fuzzy initial value problem:

$$\tilde{y}'(t) = -\tilde{y}(t)$$
$$\tilde{y}(0) = (0.96, 1.01) \qquad (5.2.1)$$

As we know the exact solution of the Differential equation (5.1.1) is:

$$y(t) = (e^{-t})c_1 \qquad (5.2.2)$$

So, the exact solution by using the fuzzy initial condition is:

$$y(t) = (0.96, 1.01)e^{-t} \qquad (5.2.3)$$

Which is obtained by solving formally the system and using fuzzy numbers in the classical solution. By using the results of Definition (2.8) the solution is a Double number valued function and the endpoints are solutions of the system:

$$\underline{y}'(t) = -\underline{y}(t)$$
$$\bar{y}'(t) = -\bar{y}(t)$$
$$\underline{y}(0) = 0.96$$
$$\bar{y}(0) = 1.01 \qquad (5.2.4)$$

Having solution:

$$\underline{y}(t) = -0.025e^{t} + 0.985e^{-t}$$
$$\bar{y}(t) = 0.025e^{t} + 0.985e^{-t} \qquad (5.2.5)$$

By using Adams-Bashforth-Moulton three-step method with *N=10*, the following results are obtained:

Table (5.2.1): Numerical solution of Example (5.2) by Adams-Bashforth-Moulton three-step method

T	$\underline{y}(t)$	$\underline{y}(t)$ *exact*	*Error*	$\bar{y}(t)$	$\bar{y}(t)$ *exact*	*Error*
0	0.96	0.96	0	1.01	1.01	0
0.1	0.86860000	0.86864392	4.3921e-005	0.91390000	0.91388579	1.4208e-005
0.2	0.7860000	0.7859815	1.8477e-005	0.82690000	0.82691806	1.8061e-005
0.3	0.71120000	0.71118549	1.4508e-005	0.74820000	0.74822640	2.6403e-005
0.4	0.64352437	0.64350724	1.7131e-005	0.6770037	0.6770232	1.9462e-005
0.5	0.58228863	0.58226943	1.9197e-005	0.61258225	0.61259596	1.3707e-005
0.6	0.52687987	0.52685917	2.0703e-005	0.55429085	0.55429975	8.8970e-006
0.7	0.47674364	0.47672189	2.1749e-005	0.50154627	0.50155115	4.8779e-006
0.8	0.43137821	0.43135580	2.2408e-005	0.45382071	0.45382225	1.5430e-006
0.9	0.39032961	0.39030687	2.2744e-005	0.41063655	0.41063535	1.2013e-006
1	0.35318707	0.35316426	2.2814e-005	0.37156167	0.37155823	3.4373e-006

And by using Adams-Bashforth-Moulton four-step method with *N=10*, the following results are obtained:

Table (5.2.2): Numerical solution of Example (5.2) by Adams-Bashforth-Moulton four-step method

T	$\underline{y}(t)$	$\underline{y}(t)$ *exact*	*Error*	$\bar{y}(t)$	$\bar{y}(t)$ *exact*	*Error*
0	0.96	0.96	0	1.01	1.01	0
0.1	0.86860000	0.86864392	4.3921e-005	0.91390000	0.91388579	1.4208e-005
0.2	0.7860000	0.7859815	1.8477e-005	0.82690000	0.82691806	1.8061e-005
0.3	0.71120000	0.71118549	1.4508e-005	0.74820000	0.74822640	2.6403e-005
0.4	0.64352003	0.64350724	1.2787e-005	0.6769991	0.6770232	2.4049e-005
0.5	0.58228082	0.58226943	1.1397e-005	0.61257387	0.61259596	2.2088e-005
0.6	0.52686923	0.52685917	1.0068e-005	0.55427949	0.55429975	2.0253e-005
0.7	0.47673078	0.47672189	8.8925e-006	0.50153260	0.50155115	1.8554e-005
0.8	0.43136365	0.43135580	7.8499e-006	0.45380525	0.45382225	1.6995e-005
0.9	0.39031379	0.39030687	6.9250e-006	0.41061979	0.41063535	1.5565e-005
1	0.35317036	0.35316426	6.1050e-006	0.37154398	0.37155823	1.4253e-005

Now we compare the results of Table (5.2.1) by using Adams-Bashforth-Moulton three-step method with the results of Table (5.2.2) by using Adams-Bashforth-Moulton four-step method, the results with *N=10* are obtained:

Table (5.2.3): Comparison the results of Adams-Bashforth-Moulton three-step method with the results of Adams-Bashforth-Moulton four-step method

T	$\underline{y}(t)3-step$	$\underline{y}(t)4-step$	*Error*	$\bar{y}(t)3-step$	$\bar{y}(t)4-step$	*Error*
0	0.96	0.96	0	1.01	1.01	0
0.1	0.86860000	0.86860000	0	0.91390000	0.91390000	0
0.2	0.7860000	0.7860000	0	0.82690000	0.82690000	0
0.3	0.71120000	0.71120000	0	0.74820000	0.74820000	0
0.4	0.64352437	0.64352003	4.3438e-006	0.6770037	0.6769991	4.5868e-006
0.5	0.58228863	0.58228082	7.8005e-006	0.61258225	0.61257387	8.3808e-006
0.6	0.52687987	0.52686923	1.0635e-005	0.55429085	0.55427949	1.1356e-005
0.7	0.47674364	0.47673078	1.2856e-005	0.50154627	0.50153260	1.3676e-005
0.8	0.43137821	0.43136365	1.4558e-005	0.45382071	0.45380525	1.5452e-005
0.9	0.39032961	0.39031379	1.5819e-005	0.41063655	0.41061979	1.6766e-005
1	0.35318707	0.35317036	1.6709e-005	0.37156167	0.37154398	1.7690e-005

CONCLUSION

We presented definitions and theorems for the solutions of the fuzzy Differential equations whish allow us to translate a FDE into a system of ordinary Differential equations. Also, a predictor-corrector methods can be used to solved FDEs the Adams-Bashforth *m*-step method and Adams-Moulton *m*-step method, also, we can be used the Adams-Bashforth *(m-1)*-step method and Adams-Moulton *(m-1)*-step method to solved FDEs, these methods are considered as predictor and corrector, respectively. For future research we can apply the predictor-corrector methods for systems and for the partial Differential equations in the fuzzy setting, Also, we can apply these methods for solving the stiff problems.

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