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A Graph Theory Analysis on the Elements of Triaxial Template of *Tudung Saji*

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ABSTRACT

An analysis of the triaxial patterns of *tudung saji* weaving has resulted in the creation of a two dimensional template which represents the planar pattern of *tudung saji*. This two dimensional template has produced many patterns including some of the original patterns found on the three dimensional *tudung saji* where the number of strands used started with two up to six. Furthermore, iMac Grapher software has been manipulated to produce several graphs based on the elements of triaxial template, according to their patterns. In this paper, a graph theory analysis is used to determine the properties of the graph produced by the iMac Grapher.

Keywords: triaxial template, planar pattern on *tudung saji*, graph theory

INTRODUCTION

Traditional crafts have played an important role in Malaysian culture. It is a way people show their ideas and creativity through the diverse use of plants and natural elements. The skills involved in producing these unique and beautiful crafts are passed down through generations.

A traditional craft that is highlighted here is *tudung saji* which is produced in Melaka and Terengganu. Basically, *tudung saji* is a food cover used mostly by the Malays to cover their meals. Since *tudung saji* can be made in various sizes, many people nowadays also use these *tudung saji* as home decorations. Previous research has been done by Adam with regard to the creation of *tudung saji* (Adam, 2011).

A food cover is woven by using a triaxial weave technique, where the strands are plaited in three directions. To form a conical shape of food cover, weavers need to form a framework (called *rangka*) first, before they insert other strands as its patterns.

Firstly, a group of five strands of dried leaves are interlaced with each other which built in a pentagonal opening as the peak of the cover. After that, another five strands are interlaced to form the hexagonal opening for the side of the cover. Apparently, a conical shape will form as they continue to interlace the strands. After a framework is finished, the insertion step will begin which eventually formed the patterns for each food cover. In addition, since the strands of dried leaves could be coloured, many beautiful and unique patterns have been created with variety of colours. Some examples of the patterns are named as Flock of Pigeons, Sailboats, Crab Fingers and Cape Flower, as shown in Figure 1.

Figure 1: Examples of food cover patterns: from left to right: Flock of Pigeons, Sailboats, Crab Fingers and Cape Flower

Previously, Adam (2011) developed a template from a set of tumbling block graph paper, named as triaxial template which is used to simulate the naturally-occurring three dimensional *tudung saji* patterns. This triaxial template is built in a triangle shape, consisting three axes, A, B, and C which intersects at 120^0. In addition, an arrow is drawn at the initial of each strand to indicate the colour insertion for the template.

Since the patterns of the original food cover are formed from variety of colours, the template's strands are also coloured to produce the patterns. Here, two vibrant colours, namely red and yellow are used. These colours are inserted into each of the arrows, started with partition A, B until C, with counter clockwise direction. To form variety of patterns, every template is coloured with different ordering, which is defined as labelling system. This labelling system is very important as it acts to create the elements of the patterns that existed from the template. An example of a triaxial template is shown in Figure 2.

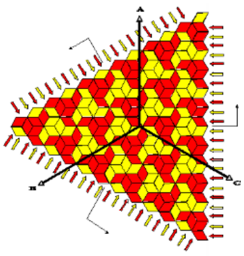


Figure 2: An example of triaxial template with Flock of Pigeons pattern

However, since this triaxial template is planar, only several of the actual *tudung saji* patterns can be simulated. Furthermore, three dimensional graphs that are related with the elements of the template were generated using the iMac Grapher software, according to their patterns (Adam, 2011).

In this research, the patterns of *tudung saji* are analyzed using graph theory. A graph G is an ordered triple set of $(V(G), E(G), \psi_G)$ which consists of a nonempty set $V(G)$ of vertices, a set $E(G)$ of edges, and an incidence function ψ_G that associates with each edge of G, an unordered pair of vertices of G (Bondy and Murty, 1976). A graph is simple if it has no loops and no two of its links join the same pair of vertices (Bondy and Murty, 1976). In addition, a graph is connected if there exists a path between every pair of distinct vertices (Singh, 2010) and it is called complete if every vertex is connected to every other vertex (Chartrand et al., 2011). A cycle in a graph is a closed path (Chartrand et al., 2011) and a graph is called acyclic if it contains no cycle (Bondy and Murty, 1976). Furthermore, a tree is a connected acyclic graph and a spanning tree of a graph is a spanning subgraph of a graph that is tree (Bondy and Murty, 1976). The chromatic number of a graph is the smallest number required to colour the vertices where adjacent vertices are different colours (Chartrand et al., 2011). A set V of vertices in G is called a clique in G if there is an edge in G between every pair of vertices in V, and the cardinality of a largest clique in G is the clique number (Balakrishnan, 1997).

SYMMETRICAL CHARACTERISTICS OF TRIAXIAL TEMPLATE PATTERNS AND GRAPHS PRODUCED ON IMAC GRAPHER SOFTWARE

From the previous study, Adam has developed several graphs based on the elements of the patterns appeared on the triaxial template (Adam, 2011). The three dimensional graphs are created by connecting the elements of the patterns according to their number of strands.

In this paper, the number of strands is referred as the number of colours inserted for each arrow, before it is repeated. The number of strands used for the labelling system in this paper started with two up to six. Hence, for the 2-strand template, 0 is used to denote red and yellow colour while 1 is used to denote yellow and red colour. For instance, 0 is equal to RY and 1 is equal to YR, where R and Y here stand for red and yellow, respectively. As for the 3-strand template, the elements are labelled as 0 is equal to RRY, 1 is equal to RYR, and 2 is equal to YRR. This labelling system is continuously applied up to 6-strand template. Since the template consists of three axes, the elements that came up from the labelling system will be the three digit sequence of 0, 1, 2, and so on.

Previously, Zamri *et al.* (2013) have described some symmetrical characteristics of triaxial patterns of 2 and 3-strand template, as shown in Table 1. In this research, the graph theory analysis is done only on the patterns based on (Zamri, *et al*., 2013).

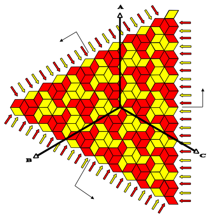
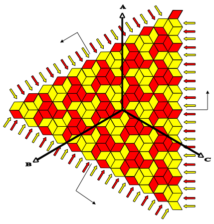
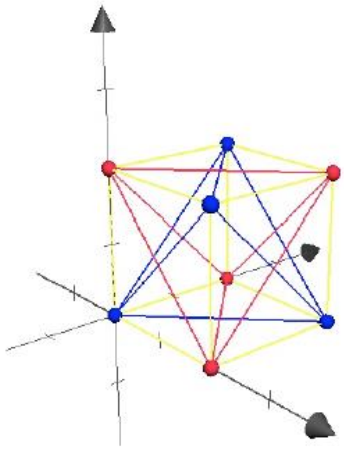
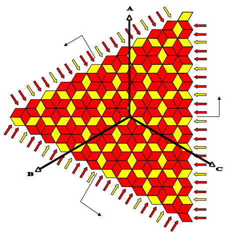
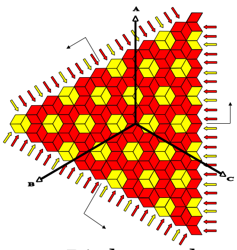
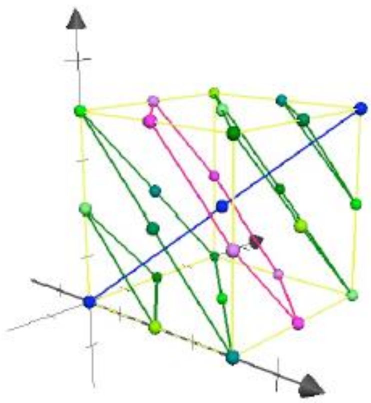
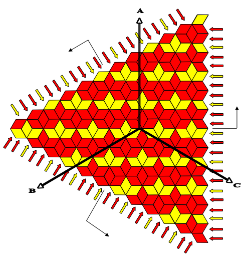
Table 1: The Elements in Each Pattern and Their Symmetrical Characteristics

No. of strand	**Name of pattern**	**Elements**	**No. of elements**	**Symmetry operation**	**Symmetry element**
2	Flock of Pigeons	{(000),(111)}	2	Inversion at a point	Center of inversion
3	Sailboats	{(010),(110),(100),(101), (001),(011)}	6	Rotation	Rotation axis
3	Buttons	{(012),(021)}	2	Reflection	Mirror plane
3	Cape Flower	{(111),(000),(222)}	3	Rotation	Rotation axis

Table 1 indicates that the elements of the Sailboats and Cape Flower patterns possessed a rotation operation over a line, while the elements of Flock of Pigeons and Buttons patterns carry an inversion at a point and reflection operations, respectively.

Based on the elements and symmetrical properties of the 2 and 3-strand template's patterns, the properties of their graphs are presented in this paper by using some properties in graph theory. In Table 2, the patterns of 2 and 3-strand triaxial template, together with their graphs developed by using iMac Grapher software are first given (Adam, 2011).

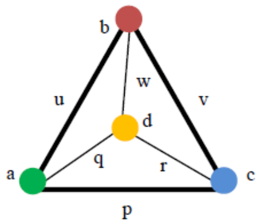
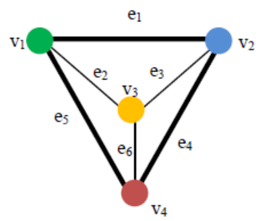
Table 2: 2 and 3-Strand Triaxial Template Patterns with Its Graphical Representations

No. of strand	Patterns	Triaxial Templates	Graphical Representations
2	Flock of Pigeons	 Blue graph  Red graph	
3	Cape Flower	 Blue graph	
	Buttons	 Pink graph	
	Sailboats	 Green graph	

MAIN RESULTS

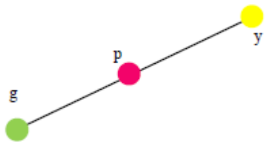
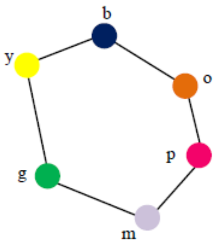
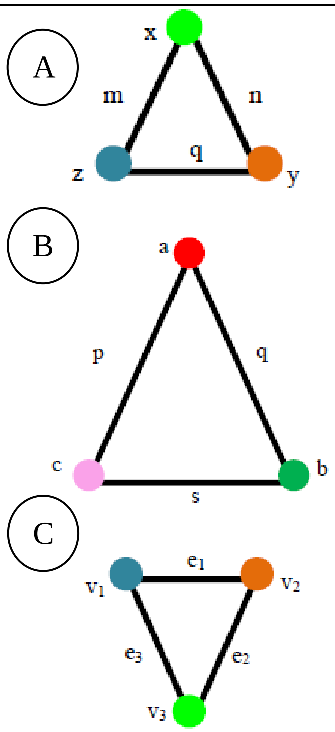
In this paper, some characteristics of the graphs representation of the 2 and 3-strand templates are analyzed according to their patterns. Besides, the isomorphisms between two graphs are also described. As given by (Bondy and Murty, 1976), two graphs G and H are isomorphic $(G \cong H)$ if there are bijections $\theta : V(G) \to V(H)$ and $\phi : E(G) \to E(H)$ such that $\psi_G(e) = uv$ if and only if $\psi_H(\phi(e)) = \theta(u)\theta(v)$ (a homomorphism). A pair (θ, ϕ) is called an isomorphism. The characteristics of graphs and their connections with graph theory are stated in Table 3 and Table 4. Table 3 depicts the analysis of the elements of the graphs for 2-strand template, which is also named as Flock of Pigeons pattern. Since this pattern has two graphs, it has been divided into two parts, namely Flock of Pigeons A and Flock of Pigeons B.

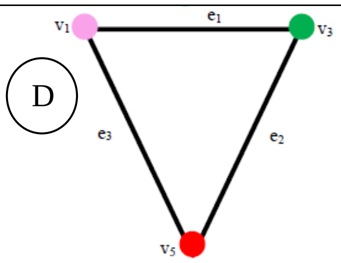
Table 3: Analysis of 2-Strand Triaxial Template Graphs with Graph Theory

Patterns	Graph	Characteristics
Flock of Pigeons A		i. A connected graph ii. A complete graph with four vertices, K_4. iii. Clique subgraphs are: K_2, K_3, K_4. iv. Clique number: $\omega(K_4) = 4$. v. Chromatic number: 4
Flock of Pigeons B		i. A connected graph ii. A complete graph with four vertices, K_4. iii. Clique subgraphs are: K_2, K_3, K_4. iv. Clique number: $\omega(K_4) = 4$. v. Chromatic number: 4
Isomorphism between Flock of Pigeons A and Flock of Pigeons B		We denote two mappings (θ, ϕ) as $\theta(a) = v_1$, $\theta(b) = v_4$, $\theta(c) = v_2$, $\theta(d) = v_3$, and $\phi(u) = e_5$, $\phi(v) = e_4$, $\phi(w) = e_6$ $\phi(p) = e_1$, $\phi(q) = e_2$, $\phi(r) = e_3$. This gives a bijection mapping. It can be shown that it is also a homomorphism. Therefore, Flock of Pigeons A and Flock of Pigeons B are isomorphic, i.e. A $\cong$ B.

From the analysis, it is shown that both graphs of Flocks of Pigeons A and B possessed the same characteristics. They have portrayed the properties of a connected and complete graph with four vertices, which is named as K_4. By introducing the mapping (θ,ϕ) of elements of Flock of Pigeons A and B, it seems that both graphs are isomorphic to each other. Since they hold the same number of strand with the same pattern, the characteristics of these two graphs are similar. In Table 4, the analysis of the elements of the graphs for 3-strand template is presented. It is found that the 3-strand template consists of three graphs with different patterns, namely Cape Flower, Buttons and Sailboats.

Table 4: Analysis of 3-Strand Triaxial Template Graphs with Graph Theory

Patterns	**Graph**	**Characteristics**
Cape Flower		i. Contains no cycle: acyclic graph ii. Connected: Yes iii. A tree iv. Has a spanning tree
Buttons		i. A graph with six vertices. ii. Since not all vertices are adjacent to each other, this is not a complete graph. iii. $\text{Dim}(K_6)=1$, since all vertices are connected.
Sailboats		i. A and C are complete and connected graph with three vertices, i.e. K_3 . ii. B and D are complete and connected graph with three vertices, i.e. K_3 . Now, we will show the isomorphism between A and C, also an isomorphism between B and D. i. A and C The pair of mappings (θ,ϕ) defined by $\theta(x)=v_3,\ \theta(y)=v_2,\ \theta(z)=v_1$ and $\phi(m)=e_3,\ \phi(n)=e_2,\ \phi(q)=e_1$ is an isomorphism between the graphs A and C. The graphs A and C clearly have the same structure, and differ only in the orientation. Therefore, A $\cong$ C.

		ii. B and D The pair of mappings (θ,ϕ) defined by $\theta(a)=v_5$, $\theta(b)=v_3$, $\theta(c)=v_1$ and $\phi(p)=e_3$, $\phi(q)=e_2$, $\phi(s)=e_1$ is an isomorphism between the graphs B and D. The graphs B and D clearly have the same structure, and differ only in the orientation. Therefore, B $\cong$ D.

From the analysis, it depicts that Cape Flower graph consists of three vertices without any cycle. Since all of its vertices are connected, it can be defined as a tree. Therefore, this graph is called acyclic graph, which also contains a spanning tree.

For Buttons graph, it is found that not all of its six vertices are adjacent to each other. Thus, this graph is not a complete graph since it is not connected. Hence, this graph can only be defined as a simple graph with six vertices.

The last graph is the graph of Sailboats pattern. This pattern is seen to hold the highest number of graphs which is four. Hence, the graphs are named as graph A, B, C and D. However, all these four graphs contain the same number of vertices which is three. Since all four graphs hold the characteristics of a complete and connected graph, we can define them as K_3. Furthermore, by defining the mapping (θ,ϕ) of elements between graph A and C, and graph B and D, we can finally define their isomorphisms.

CONCLUSION

Based on the analysis of the characteristics of the graphs of 2 and 3-strand triaxial templates, it is found that they hold three properties of graphs which are connected and complete graph, acyclic graph and also simple graph. Besides, the graphs with the same patterns are seen to be isomorphic to each other as shown by Flock of Pigeons and Sailboats graphs. Therefore, for future research, the analysis of the graphs of triaxial template up to six numbers of strands could be done to get the general patterns of the graphs.

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REFERENCES

Adam, N. A. (2011), *Weaving Culture and Mathematics*, Ph. D. Thesis, The University of Auckland.

Bondy, J. A. and Murty, U. S. R. (1976), *Graph Theory with Applications*. 1^{st} Published in Macmillan Press Ltd.

Singh, G. S. (2010), *Graph Theory*. New Delhi: PHI Learning Private Limited.

Chartrand, G., Lesniak, L. and Zhang, P. (2011), *Graphs and Digraphs*, *5th ed*. Rutgers School of Arts and Science.

Balakrishnan, V. K. (1997), *Schaum's outlines of theory and problems of graph theory*. United States: The McGraw-Hall Companies.

Zamri, S. N. A., Sarmin, N. H., Adam, N. A. and Mohd Sani, A. (2013), Modelling *Tudung Saji* Weaving Using Group Theory. *In Proceedings of the International Graduate Conference on Engineering, Science and Humanities* (IGCESH2013), Universiti Teknologi Malaysia, April. 16-17, pp. 917-920.