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MHD Influences on Non-Newtonian Blood Flow Through a Multiple Stenosed Artery

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ABSTRACT

A two dimensional generalized Newtonian fluid is consider in order to analyze the effect of a uniform transverse magnetic field through the artery in the presence of multiple stenoses. An updated numerical scheme for solving the governing unsteady magnetohydrodynamics (MHD) equations quantitatively is developed. The finite difference approximations based on the well-known staggered grids for the Marker and Cell (MAC) method are used to discretize the governing equations of the boundary value problem under study. The graphical representations of the blood characteristics like the velocity, the wall shear stress and the streamlines are well presented at the end of the paper. The results show that the flow separates mostly towards downstream of the multiple stenoses. However, the flow separation regions keep on shrinking with the enhancement of the magnetic-field and eventually it completely disappears for large magnetic-field.

Keywords: Magnetohydrodynamics, Non-Newtonian flow, Flow separation, Shear-dependent viscosity

INTRODUCTION

Studies on magneto-hydrodynamics (MHD) effects are very interesting and important on the biological systems and clinical applications. The well known studies in this relevant domain are the magnetic drug targeting for transporting drugs to the whole human body and the development of magnetic devices for cell separation (Voltairas *et al.* (2001) and Haik *et al.* (1999)). The magnetic susceptibility of blood was found to be 3.5×10^{-6} and -6.6×10^{-7} for the oxygenated and deoxygenated, respectively and by using a higher magnetic field (10 Tesla), blood flow rate decreases by 30% (Haik *et al.* (1999)). Ichioka *et al.* (2000) used strong magnetic fields (8 Tesla) on a living rat and they showed that both the blood flow and the temperature decrease significantly during magnetic field exposure. By Lenz's law, the Lorentz force arising out of the fluid and since blood is an electrically conducting fluid, the changes of the hemodynamic factors of blood flow in human arteries are important for the cardiovascular system Midya *et al.* (2003). Midya *et al*. (2003) showed that the flow separation can be prevented by the application of the magnetic field.

Barnothy (1964) reported that the heart rate decreases considerably when the biological systems are exposed to an external magnetic-field while Vardanyan (1973) investigated the potential use of MHD principles in prevention and rational therapy of arterial hypertension and found that for steady flow of blood in an artery of circular crosssection, a uniform transverse magnetic-field alters the flow rate of blood. Rao and Desikachar (1986) explored the effect of a magnetic field on

MHD oscillatory flow of blood oxygenation in a channel of varying cross-section and the results obtained from their investigation showed that the biological systems are greatly influenced by the application of external magnetic-field. Recently, Ramamurthy and Shanker (1994) studied the magnetohydrodynamic effects on blood flow through a porous channel. The steady MHD flow of an electrically conducting viscous fluid in slowly varying channel in the presence of a uniform transverse magnetic field was also studied by Pal *et al.* (1996). Rathod *et al* (2006) found from their study on the pulsatile blood flow subject to periodic body acceleration and magnetic-field that the velocity is decreased considerably when the magnetic intensity is allowed to increase.

Atherosclerosis is an arterial disease caused by accumulative plaques leading to the malfunction of the cardiovascular system. Such plaques or arterial stenoses disturb normal blood flow through the artery which will leading to herat attack. Since coronary artery disease may have multiple sites of narrowing, some attention has been devoted to examining the influence of multiple stenoses on coronary flow, both experimentally and numerically. The studies relating to blood flow in the presence of two stenoses arranged in series has been successfully carried out by several researchers (Mustapha *et al.* (2008, 2009, 2011), Dreumel and Kueken (1989), Kilpatrick *et al.*(1990), Johnston and Kilpatrick (1990)).

In most of the previous studies, the flowing blood is assumed to be Newtonian. However, from experimental studies, it is observed that blood behaves like a non-Newtonian fluid at low shear rates ($0.1s^{-1}$) and in vessels of smaller diameter (Ku(1997), Deshpande *et al.* (1976)) and exhibits marked shear thinning and significant viscoelastic properties in pulsatile flows (Philips and Deutch(1975), Oiknine (1983)). Yeleswarapu (1996) modeled blood as a generalized Newtonian fluid and includes a shear rate dependent viscosity and has been proved effective to describe blood flows in the typical range of shear rates once a range for the material constants is determined. Recently, studies on generalized Newtonian model for blood past straight small vessels have been successfully carried out by Pontrelli (2000, 2001) and Sariffudin *et al.* (2007).

The objective of this study is to investigate the influence of MHD to the non-Newtonian fluid flow and to explore the effects of multiple stenoses on the flow characteristics of blood. The Marker and Cell (MAC) method (Harlow and Welch (1965)) based on staggered grid has been used to solve the unsteady generalized Newtonian equations in cylindrical polar co-ordinates system.

STENOSES MODEL

The geometry of the stenosis considered herein is multiple stenoses with 60% areal occlusion as shown in Figure 1.

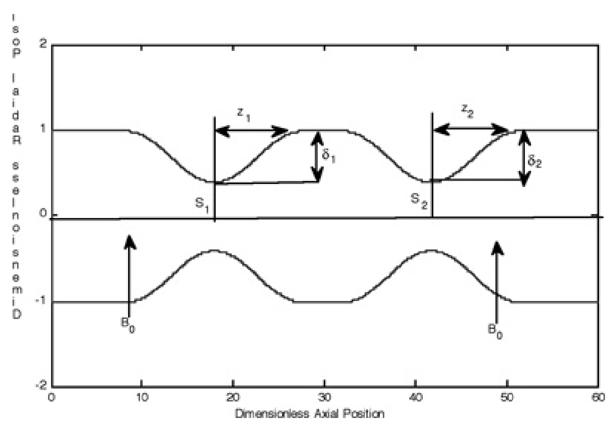

Figure 1: Profile of multiple stenoses with the presence of a uniform magnetic field, B_0.

Denoting $R\ (z,\ t)$ as the time-dependent radius of the artery in stenotic region given by

$$(z,t)=\begin{cases} a_1(t)\left[1-\dfrac{\delta_1}{2R_0}\left(1+\cos\left(\dfrac{\pi(z-S_1)}{Z_1}\right)\right)\right], S_1-Z_1 z S_1+Z_1 \\ a_1(t)\left[1-\dfrac{\delta_2}{2R_0}\left(1+\cos\left(\dfrac{\pi(z-S_2)}{Z_2}\right)\right)\right], S_2-Z_2 z S_2+Z_2 \\ a_1(t), \qquad\qquad \text{otherwise}\end{cases} \tag{1}$$

where R_0 is the radius in non-stenotic region. The time-variant parameter is described as

$$a_1(t)=1+k_R\cos(\omega t-\phi) \tag{2}$$

in which $\omega=2\pi f_p$ is the angular frequency with f_p is the pulse frequency and k_R is a constant.

GOVERNING EQUATIONS

Let us consider the stenotic blood flow in the stenosed artery to be two-dimensional flow of an incompressible viscous fluid of density ρ. The flow is assumed to be axisymmetric and fully developed characterized by the generalized Newtonian model. The Navier-Stokes equations governing the fluid motion and the equation of continuity may be written in cylindrical co-ordinate system as

$$\begin{aligned}&\frac{\partial w}{\partial t}+\frac{\partial(wu)}{\partial r}+\frac{\partial w^2}{\partial z}+\frac{(wu)}{r}=-\frac{\partial p}{\partial z}+\frac{\mu(\dot\gamma)}{\mathrm{Re}}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right)\\&+\frac{1}{\mathrm{Re}}\left[\frac{\partial\mu}{\partial r}\left(\frac{\partial u}{\partial r}+\frac{\partial w}{\partial r}\right)+2\frac{\partial\mu}{\partial z}\frac{\partial w}{\partial z}\right]-\frac{M^2}{\mathrm{Re}}w,\end{aligned} \tag{3}$$

$$\begin{aligned}&\frac{\partial u}{\partial t}+\frac{\partial u^2}{\partial r}+\frac{\partial(wu)}{\partial z}+\frac{u^2}{r}=-\frac{\partial p}{\partial r}+\frac{\mu(\dot\gamma)}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right)\\&+\frac{1}{\mathrm{Re}}\left[2\frac{\partial\mu}{\partial r}\frac{\partial u}{\partial r}+\frac{\partial\mu}{\partial z}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)\right],\end{aligned} \tag{4}$$

$$r\frac{\partial w}{\partial z}+\frac{\partial(ur)}{\partial r}=0, \tag{5}$$

where w and are u the axial and the radial velocity components, respectively, p is the dimensionless pressure, Re is the Reynolds number, M is the Hartmann number, $M=B_0 r_0\sqrt{\sigma/\mu}$ and $\mu(\dot\gamma)$ represents the viscosity of blood where $p=\dfrac{p'}{U_0^2\rho}$, $\mathrm{Re}=\dfrac{U_0R_0\rho}{\eta_\infty}$, U_0 is the cross-sectional average velocity and

$$\mu(\dot\gamma)=1+(\lambda-1)\left[\frac{1+\log_e(1+\Lambda\dot\gamma)}{1+\Lambda\dot\gamma}\right], \tag{6}$$

as suggested by Yeleswarapu (1996), with

$$\dot{\gamma} = \left[2\left(\frac{\partial u}{\partial r}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + 2\left(\frac{u}{r}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)^2\right]^{\frac{1}{2}} \tag{7}$$

The non-dimensional viscosity parameters $\lambda = \frac{\eta_0}{\eta_\infty}, \Lambda \to \frac{\wedge U_0}{r_0}$, with η_0 and $\eta_\infty(\eta_0 \geq \eta_\infty)$ the asymptotic apparent viscosities as $\dot{\gamma} \to 0$ and ∞ respectively, and $\Lambda \geq 0$ is a material constant with the dimension of time representing the degree of shear-thinning. This model reduces to the Newtonian one for $\eta_0 = \eta_\infty$ i.e for so that $\mu(\dot{\gamma})$ becomes constant. The present approximation of a three-parameter shear-thinning model characterizing the complexity in blood rheology indicates that the apparent viscosity as a decreasing function of the shear rate which increases considerably at low shear rates.

Boundary and Initial Conditions

According to Mustapha *et al* (2009), the boundary conditions are taken as:

$$w(r,z,t) = 0, \;\; u(r,z,t) = \frac{\partial R}{\partial t} \;\; \text{for} \;\; r = R(z,t) \tag{8}$$

$$\frac{\partial w(r,z,t)}{\partial r} = 0, \;\; u(r,z,t) = 0 \;\; \text{for} \;\; r = 0 \tag{9}$$

$$w(r,z,t) = 2\left(1 - \frac{r^2}{R^2}\right), \;\; u(r,z,t) = 0 \;\; \text{for} \;\; z = 0, M = 0 \tag{10}$$

$$w(r,z,t) = 2\left(\frac{\mathrm{I}0(MR)}{\mathrm{I}0(MR) - 1}\right)\left(1 - \left(\frac{\mathrm{I}0(Mr)}{\mathrm{I}0(MR)}\right)\right), \;\; u(r,z,t) = 0 \;\; \text{for} \;\; z = 0, M \neq 0, \tag{11}$$

$$\frac{\partial w(r,z,t)}{\partial z} = 0 = \frac{\partial u(r,z,t)}{\partial z} \;\; \text{for} \;\; z = L, \tag{12}$$

where I_0 is a Bessel function. In the downstream and L being the dimensionless length of the arterial segment under consideration. The initial conditions given by:

$$w(r,z,0) = 0, \;\; u(r,z,0) = 0\;, \; p(r,z,0) = 0 \;\; \text{for} \;\; z > 0 \tag{13}$$

NUMERICAL RESULTS AND DISCUSSIONS

The complete numerical algorithm already discussed in Mustapha *et al* (2008,2009,2011). For the purpose of numerical computation of the desired quantities of major physiological significance, the following parameters have been ranged around some typical values in order to obtain results of physiological interest (Yeleswarapu(1996), Pontrelli (2000, 2001) and Sariffudin *et al.* (2007)):

$$k_R = 0.0001, \omega = 2\pi f_p, f_p = 1.2Hz, \phi = 0^0, \rho = 1.05 \times 10^3 kgm^{-3},$$
$$R_0 = 0.154cm, \Delta x = 0.025, \lambda = 5, \Lambda = 50.$$

The numerical results present in this section are obtained after the steady state is achieved. In order to validate the applicability of the model under consideration, we already compare our numerical results with the experimental results and the results presented in Mustapha *et al* (2009). The present study is the extension of Mustapha *et al* (2009). The numerical results for present problem with the velocity, wall shear stress and streamlines are describe in Figures 2-4.

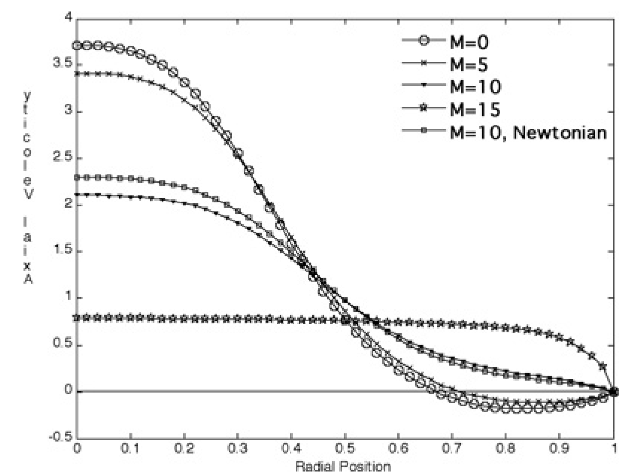

Figure 2: The axial velocity for different values of M at z=100 and Re=300.

Figure 2 illustrates the axial velocity of the blood through a multiple stenoses at the critical point of the stenosis (z = 17.5). As we can see that the velocities in a tube are decrease with the increasing of the magnetic fields. However, the velocities become higher when approach to the wall. Besides, in the figure, when M=0 and M=5, the backflows clearly observed, but the backflows dissmiss when M increase. The backflow is recognized when the velocity changes from positive to negative. On the other hand, the figure also shows that when comparing with the Newtonian fluid, the velocity for the generalized Newtonian model is lower.

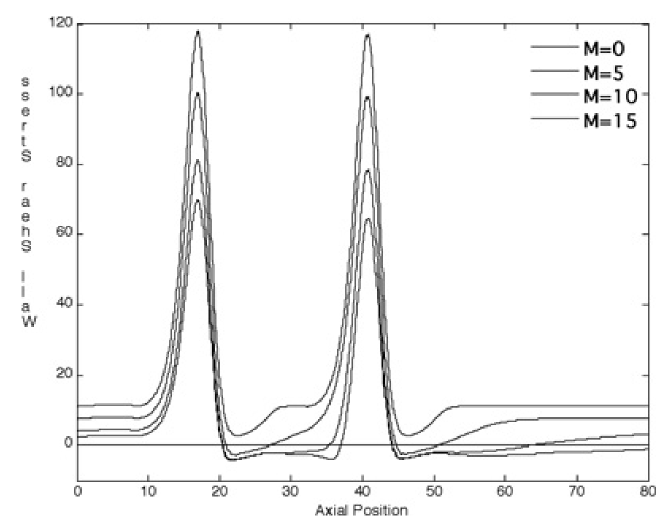

Figure 3: Variations of wall shear stress at Re=300

Figure 3 shows the variations of wall shear stress for different values of magnetic fields at Re=300. The narrowest passage of the outlines of the stenoses give rise to higher wall shear stress in the converging region of the stenosis as portrayed in the figure. The wall shear stress for generalized Newtonian fluids increase as the values of the magnetic field increase. It is interesting to note that at this value of Reynolds number, the flow separations form at the downstream of the stenoses with absent magnetic field. But we can see clearly that the flow separations reduce when the magnetic fields exist and it totally disappear with higher magnetic field value, which is M=15. This is may due to the reason where the existing of the magnetic fields give rise to the higher values of the flow velocities near the wall that can prevent the flow from become separates.

Figure 4 displays the instantaneous patterns of streamlines governing the flow of blood through the multi-irregular stenoses at Re = 300 with various values of magnetic fields. It is clearly seen that a large recirculation region develops at the downstream of more the stenoses with absence of magnetic field. For both stenoses, it can be seen that the recirculation regions develop at two places, which are in between the two stenoses and in the diverging section of the second stenosis. The streamlines panels show that more and larger recirculation regions are noted in the case of no magnetic field and it diminishes completely when M=15.

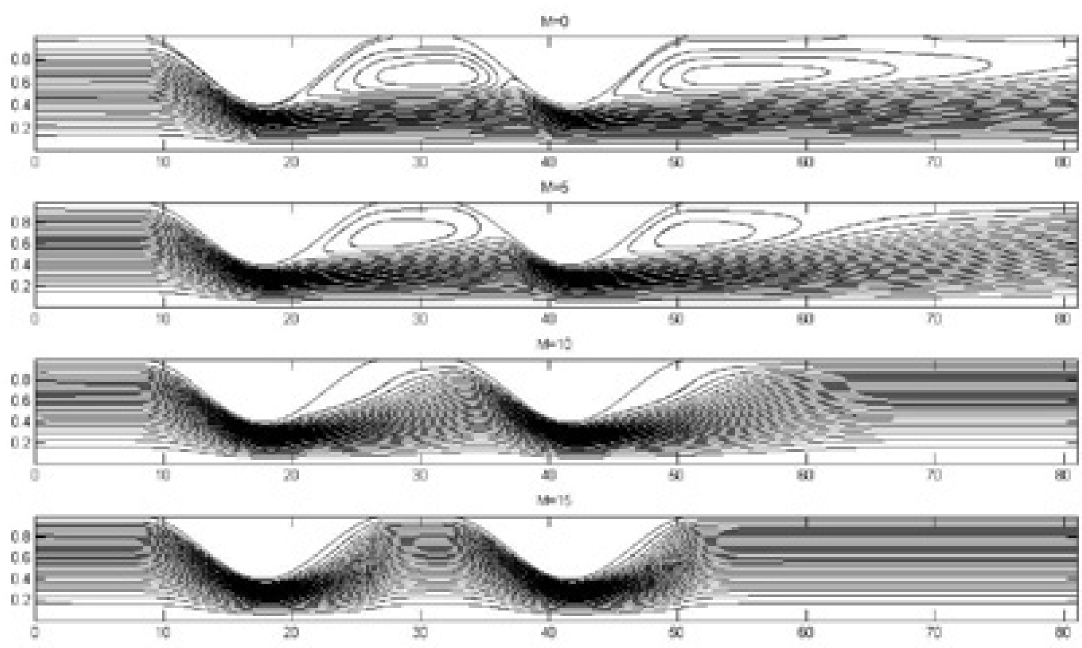

Figure 4: Instantaneous patterns of streamlines for different values of M at Re=300

CONCLUSION

A two-dimensional axisymmetric mathematical model to study the characteristics of generalized Newtonian blood flow through an artery in the presence of multi-stenoses with the magnetic field effects has been developed and solved by the MAC method. The characteristics of blood flow such as velocity and wall shear stress are investigated and have been presented in graphical representations (Figures 2 – 4). By considering the generalized Newtonian rheology of blood with shear dependent viscosity in the realm of the presence of blood cells and plasma in blood. Besides, this present study which also concerned with flow unsteadiness and wall flexibility is found to improve from the previous studies in researches of blood flow through a constricted artery.

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