

**SKEMA PENYELESAIAN
BAHAGIAN A
(12 Markah)**

SOALAN A1

BM Seorang lelaki memandu pada kelajuan 90 km/j. Apabila menyedari dia telah terlewat, dia meningkatkan kelajuannya kepada 110 km/j dan melengkapkan perjalanannya sejauh 395 km dalam masa 4 jam. Berapa lamakah dia memandu pada kelajuan 90 km/j?

BI *A man was driving at 90km/h. Realizing that he was late, he increased his speed to 110km/h and completed his journey of 395 km in 4 hrs. For how long did he drive at 90 km/h?*

PENYELESAIAN SOALAN A1

Let the time he drives at 90 km/hr = t and the time he drives at 110 km/h = $4 - t$

So,

$$90t + 110(4 - t) = 395$$

$$20t = 45$$

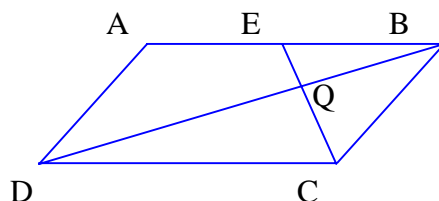
$$t = 2.25 \text{ jam atau } 2 \text{ jam } 15 \text{ minit atau } 135 \text{ minit}$$

Jawapan:	$t = 2.25$ jam atau 2 jam 15 minit atau 135 minit
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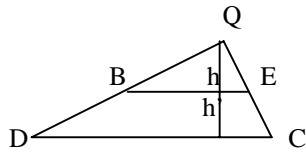
SOALAN A2

BM Misalkan ABCD satu segiempat selari dengan E satu titik di atas garis AB dengan keadaan $3BE = 2DC$. Garis CE dan garis BD bersilang di titik Q. Jika luas ΔDQC ialah 36, cari luas ΔBQE .

BI *Let ABCD be a parallelogram where E is a point on AB such that $3BE = 2DC$. The lines CE and BD intersect at the point Q. If the area of ΔDQC is 36, find the area of ΔBQE .*

PENYELESAIAN SOALAN A2

$\triangle DQC$ and $\triangle BQE$ are similar



$$\frac{BE}{DC} = \frac{2}{3} \Rightarrow \frac{h}{h'} = \frac{2}{3}$$

$$\text{Area of } \triangle DQC = \frac{1}{2}DC \times h = 36 \Rightarrow DC \times h = 72$$

$$\text{Area of } \triangle BQE = \frac{1}{2}BE \times h = \frac{1}{2} \times \frac{2}{3}DC \times \frac{2}{3}h = \frac{2}{9}DC \times h = \frac{2}{9} \times 72 = 16$$

OR

Since $\triangle DQC$ and $\triangle BQE$ are similar, and since $3BE = 2DC$,

All corresponding sides the same ratio

$$\frac{BE}{DE} = \frac{QE}{QC} = \frac{BQ}{DQ} = \frac{h}{h'} = \frac{2}{3} \therefore \frac{\text{Area } \triangle BQE}{\text{Area } \triangle DQC} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\text{Thus area of } \triangle BQE = \frac{4}{9} \text{ area } \triangle DQC = 16$$

Jawapan:	16
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SOALAN A3

BM Selesaikan $200520062007^2 - 200520062006 \times 200520062008$.

BI Solve $200520062007^2 - 200520062006 \times 200520062008$.

PENYELESAIAN SOALAN A3

Jika $t = 200520062007$ maka

$$200520062007^2 - 200520062006 \times 200520062008 = t^2 - (t-1)(t+1) = t^2 - (t^2 - 1) = 1$$

Jawapan:	1
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SOALAN A4

BM Misalkan $0 < x < 1$. Jika $A=x$, $B=x^2$, $C=\frac{1}{x}$ dan $D=\frac{1}{x^2}$ susunkan daripada nilai yang terkecil kepada yang terbesar.

BI Let $0 < x < 1$. If $A=x$, $B=x^2$, $C=\frac{1}{x}$ and $D=\frac{1}{x^2}$ arrange them from the smallest to the largest value.

PENYELESAIAN SOALAN A4

$$0 < x < 1 \Rightarrow x^2 < x$$

$$0 < x < 1 \Rightarrow \frac{1}{x} > 1$$

$$x^2 < x \Rightarrow \frac{1}{x^2} > \frac{1}{x}$$

$$\therefore 0 < x^2 < x < 1 < \frac{1}{x} < \frac{1}{x^2}$$

Therefore the arrangements are: **BACD**

Jawapan:	BACD
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SOALAN A5

BM Cari integer x, y dan z yang memenuhi $xy + \frac{1}{z} = yz + \frac{1}{x}$ dengan $x \neq z$.

BI Find integers x, y and z satisfying $xy + \frac{1}{z} = yz + \frac{1}{x}$ where $x \neq z$.

PENYELESAIAN SOALAN A5

$$xy + \frac{1}{z} = yz + \frac{1}{x} \Rightarrow (x-z)y = \frac{1}{x} - \frac{1}{z} = \frac{z-x}{xz} \Rightarrow xyz = -1.$$

Since x, y, z are integers and $x \neq z$, the only possibilities are

$$(x, y, z) = (1, 1, -1) \text{ or } (x, y, z) = (-1, 1, 1).$$

Jawapan:	$(x, y, z) = (1, 1, -1)$ atau $(x, y, z) = (-1, 1, 1)$
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SOALAN A6

BM 20 orang menggali sebuah kolam ikan selama 12 hari jika mereka bekerja selama 6 jam sehari. Berapakah bilangan hari yang diperlukan untuk menggali kolam yang sama jika 4 orang bekerja selama 5 jam sehari?

BI *20 persons dig a fish pond in 12 days if they work 6 hours per day. How many days is required to dig the same pond if 4 persons work for 5 hours per day?*

PENYELESAIAN SOALAN A6

$20 \times 6 \times 12$ is for one work done

Let x be the number of days required by 4 persons working for 5 hours per day

$$\text{So, } x = \frac{20 \times 6 \times 12}{4 \times 5} = 72 \text{ days}$$

Jawapan:	72 hari
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BAHAGIAN B
(18 Markah)

SOALAN B1

BM Dengan menggunakan angka 1, 2, 3, 6, 7, 9, 0 sahaja, tentukan angka mana yang boleh dipadankan dengan huruf di bawah supaya hasil tambahnya adalah betul.

$$\begin{array}{r} \text{P A K} \\ + \text{M A K} \\ \hline \text{P I U T} \end{array}$$

BI *Using only the numbers 1, 2, 3, 6, 7, 9, 0, find which number goes with each letter in the addition problem below to make it correct.*

$$\begin{array}{r} \text{P A K} \\ + \text{M A K} \\ \hline \text{P I U T} \end{array}$$

PENYELESAIAN SOALAN B1

Hasiltambah tiga digit ≤ 2000 jadi $P=1, M=9, I=0$.

U dan T tidak boleh 4

K tidak boleh 7 atau 2.

Mak ayang tiggal ialah 3, 6 .

Kalau $K=3$ maka $T=6$, dan $A=2$

Jika $K=6$ maka $A=3$, dan $T=2$

Note: Kaedah cuba jaya tak diterima.

ATAU

$K + K = 2K = T \Rightarrow T$ must be an even number

$T = \text{mod } 2, 6, 0$, but $T \neq \text{mod } 0$ because then $K = T$

$\therefore T = \text{mod } 2, 6 \Rightarrow K = 1, 3, 6$

1

Likewise, $2A = U$

If $T < 10$, then U is even

If $T > 10$, then U is odd

1

In both cases, $A = 1, 3, 6$

Also $P + M = I = 10(P) + I < 20$

So $P < 2$, that is, $P = 1$

1

Test for all possibilities:

$K, A \neq 1$, because $P = 1$

If $K = 3, T = 6$, then $A = 3$ or 6 which is not possible

$\therefore K = 6, T = 12 \text{ mod } 10 = 2$

$A = 3, U = 3 + 3 + 1 = 7$ because $T > 10$

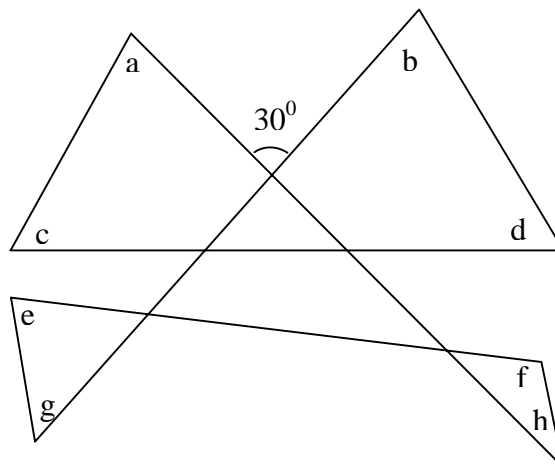
Since $P = 1$, $M = 9$ and $I = 10 \bmod 10 = 0$

$$\therefore \begin{array}{r} \text{PAK} \\ + \text{MAK} \\ \hline \text{PIUT} \end{array} \quad \begin{array}{r} 136 \\ + 936 \\ \hline 1072 \end{array} \quad \boxed{2}$$

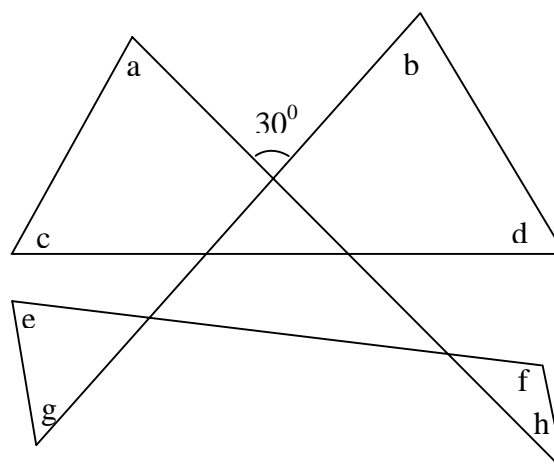
Note: Kaedah cuba jaya tak diterima.

SOALAN B2

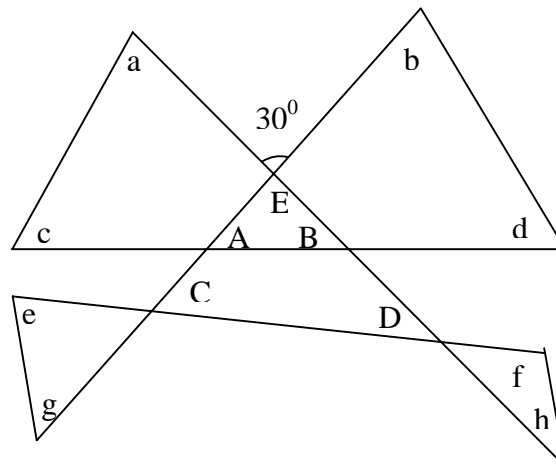
BM Cari jumlah sudut-sudut $a+b+c+d+e+f+g+h$ dalam rajah berikut.



BI Find the sum of angles $a+b+c+d+e+f+g+h$ in the following diagram.



PENYELESAIAN SOALAN B2



It is clear that $A+B+E = \pi$

$$C+D+E = \pi$$

$$B+a+c = \pi$$

$$A+b+d = \pi$$

$$C+e+g = \pi$$

$$D+f+h = \pi$$

2

Hence $\{a+b+c+d+e+f+g+h\} + \{A+B+C+D\} = 4\pi$

$$\{a+b+c+d+e+f+g+h\} + 2\pi - 2E = 4\pi.$$

Since $E = 30^\circ$, thus $\{a+b+c+d+e+f+g+h\} = 420^\circ$

4

SOALAN B3

BM Diberi $8\left(y^2 + \frac{1}{y^2}\right) - 56\left(y + \frac{1}{y}\right) + 112 = 0$ cari semua nilai $y^2 + \frac{1}{y^2}$.

BI Given $8\left(y^2 + \frac{1}{y^2}\right) - 56\left(y + \frac{1}{y}\right) + 112 = 0$ find all the values of $y^2 + \frac{1}{y^2}$.

PENYELESAIAN SOALAN B3

$$8\left(y^2 + \frac{1}{y^2}\right) - 56\left(y + \frac{1}{y}\right) + 112 = 0$$

Let $u = y + \frac{1}{y} \Rightarrow u^2 - 2 = y^2 + \frac{1}{y^2}$. 3

$$8(u^2 - 2) - 56u + 112 = 0$$

$$8u^2 - 56u + 96 = 0$$

$$u^2 - 7u + 12 = 0$$

$$(u - 3)(u - 4) = 0$$
 1

$$u = 3, 4$$

$$y^2 + \frac{1}{y^2} = 16 - 2 \quad \text{atau} \quad 9 - 2$$

$$= 14 \qquad \qquad \qquad 7$$
 2

**SKEMA PENYELESAIAN
BAHAGIAN A
(12 Markah)**

SOALAN A1

BM Diberi $x + y = 2$ dan $2xy - z^2 = 1$. Dapatkan penyelesaian integer untuk persamaan-persamaan ini.

BI Given $x + y = 2$ and $2xy - z^2 = 1$. Find the integer solutions of the equations.

PENYELESAIAN SOALAN A1

$x + y = 2$ and $2xy - z^2 = 1$ leads to $2(x - 1)^2 + z^2 = 1$, hence integer solutions are $(1,1,1), (1,1,-1)$

Jawapan:	$(1,1,1), (1,1,-1)$
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SOALAN A2

BM Dalam suatu kejohanan sukan yang berlangsung selama 4 hari, terdapat n pingat untuk dimenangi. Pada hari pertama, $1/5$ daripada n pingat dimenangi. Pada hari kedua, $2/5$ daripada baki pingat pada hari pertama dimenangi. Pada hari ketiga, $3/5$ daripada baki pingat pada hari kedua dimenangi. Pada hari keempat, 24 pingat dimenangi. Berapakah jumlah pingat kesemuanya?

BI In a sport's tournament lasting for 4 days, there are n medals to be won. On the first day, $1/5$ of the n medals are won. On the second day, $2/5$ of the remainder from the first day are won. On the third day, $3/5$ of the remainder from the second day are won. On the final day, 24 medals are won. What was the total number of medals?

PENYELESAIAN SOALAN A2

Let T_i be the number of medals won on the i th day, $i = 1, 2, 3, 4$.

Then

$$T_1 = \frac{1}{5}n,$$

$$T_2 = \frac{2}{5}(n - T_1) = \frac{8n}{25},$$

$$T_3 = \frac{3}{5}(n - (T_1 + T_2)) = \frac{36n}{125}.$$

$$T_4 = 24.$$

$$T_1 + T_2 + T_3 + T_4 = n$$

$$\frac{101n}{125} + 24 = n$$

$$\frac{24n}{125} = 24$$

$$\therefore n = 125$$

Jawapan:	n=125
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SOALAN A3

BM Misalkan $2xyz7$ suatu nombor lima angka sedemikian hingga hasil darab angka-angka tersebut ialah sifar dan hasil tambah angka-angka tersebut pula boleh dibahagikan dengan 9. Cari bilangan nombor-nombor tersebut.

BI *Let $2xyz7$ be a five-digit number such that the product of the digits is zero and the sum of the digits is divisible by 9. Find how many such numbers.*

PENYELESAIAN SOALAN A3

One of the digits must be zero. The sum of the other two digits must be divisible by 9. Possible pairs are : (0,0),(0,9),(9,9),(1,8),(2,7),(3,6),(4,5).

The total number of such numbers with the given pair :

(0,0) 1, (0,9) 3, (9,9) 3, (1,8) 6, (2,7) 6, (3,6) 6, (4,5) 6.

There are 31 such numbers

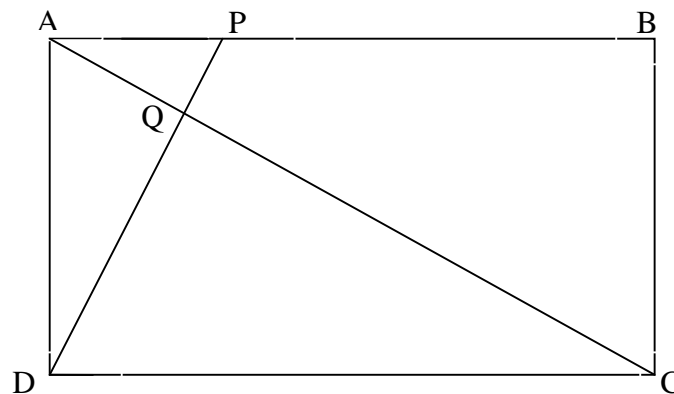
Jawapan:	31
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SOALAN A4

BM Misal ABCD sebagai suatu segiempat tepat. Garis DP memotong pepenjuru AC pada Q dan membahagikannya pada nisbah 1:4. Jika luas segitiga APQ satu unit persegi, tentukan luas segiempat tersebut.

BI *Let ABCD be a rectangle. The line DP intersects the diagonal AC at Q and divides it in the ratio of 1:4. If the area of triangle APQ is one unit square, determine the area of the rectangle.*

PENYELESAIAN SOALAN A4



Biar tinggi segitiga APQ ialah x , dan tinggi segitiga AQP ialah y . Segitiga APQ dan segitiga CDQ adalah sebangun, maka

AP: DC = AQ:QC = x : y = 1: 4. Luas segiempat DC $(x+y) = 4AP (x + 4x) = 20 AP \cdot x = 40$ (Luas segitiga APQ) = 40

Jawapan:	40
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SOALAN A5

BM Cari integer terkecil yang memenuhi syarat apabila dibahagi dengan 2 meninggalkan baki 1, apabila dibahagi dengan 3 meninggalkan baki 2, apabila dibahagi dengan 4 meninggalkan baki 3 dan apabila dibahagi dengan 5 meninggalkan baki 4.

BI Find the smallest integer such that if divided by 2 leaves a remainder of 1, if divided by 3 leaves a remainder of 2, if divided by 4 leaves a remainder of 3, and if divided by 5 leaves a remainder of 4.

PENYELESAIAN SOALAN A5

Let N be the integer. Then

$$N = 2q_1 + 1$$

$$= 3q_2 + 2$$

$$= 4q_3 + 3$$

$$= 5q_4 + 4$$

Observe that

$$N + 1 = 2q_1 + 2 = 3q_2 + 3 = 4q_3 + 4 = 5q_4 + 5$$

$$= 2(q_1 + 1) = 3(q_2 + 1) = 4(q_3 + 1) = 5(q_4 + 1)$$

$$\therefore 2|N + 1, 3|N+1, 4|N + 1 \text{ dan } 5|N+1$$

$$\Rightarrow N + 1 = \text{LCM}(2, 3, 4, 5) = 60$$

$$\therefore N = 59$$

Jawapan:	59
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SOALAN A6

BM Andaikan f suatu fungsi ditakrif pada integer sedemikian

$$f(2n) = -2f(n), f(2n+1) = f(n) - 1, \text{ dan } f(0) = 2.$$

Cari nilai $f(2007)$.

BI Let f be a function defined on integers such that

$$f(2n) = -2f(n), f(2n+1) = f(n) - 1, \text{ and } f(0) = 2.$$

Find the value of $f(2007)$.

PENYELESAIAN SOALAN A6

$$\begin{aligned} f(2007) &= f(1003) - 1 = f(501) - 2 = f(250) - 3 = -2f(125) - 3 \\ &= -2f(62) - 1 = 4f(31) - 2 = 4f(15) - 6 = 4f(7) - 10 = 4f(3) - 14 \\ &= 4f(1) - 18 = -14 \end{aligned}$$

Jawapan:	-14
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BAHAGIAN B
(18 Markah)

SOALAN B1

BM Diberi $8\left(y^2 + \frac{1}{y^2}\right) - 56\left(y + \frac{1}{y}\right) + 112 = 0$ cari semua nilai $y^2 + \frac{1}{y^2}$.

BI Given $8\left(y^2 + \frac{1}{y^2}\right) - 56\left(y + \frac{1}{y}\right) + 112 = 0$ find all the values of $y^2 + \frac{1}{y^2}$.

PENYELESAIAN SOALAN B1

$$8\left(y^2 + \frac{1}{y^2}\right) - 56\left(y + \frac{1}{y}\right) + 112 = 0$$

Let $u = y + \frac{1}{y} \Rightarrow u^2 - 2 = y^2 + \frac{1}{y^2}$. 3

$$8(u^2 - 2) - 56u + 112 = 0$$

$$8u^2 - 56u + 96 = 0$$

$$u^2 - 7u + 12 = 0$$

$$(u - 3)(u - 4) = 0$$
 1

$$u = 3, 4$$

$$y^2 + \frac{1}{y^2} = 16 - 2 \quad \text{atau} \quad 9 - 2$$

$$= 14 \qquad \qquad \qquad 7$$
 2

SOALAN B2

BM Satu sisi sebuah segitiga berukuran 4 cm. Dua sisi yang lain berukuran dalam nisbah 1:3. Cari luas yang terbesar untuk segitiga ini.

BI One side of a triangle is 4cm. The other two sides are in the ratio 1:3. What is the largest area of the triangle?

PENYELESAIAN SOALAN B2

With Heron's formula the area of the triangle with sides a, b, c is

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
 2

Where $s = \frac{a + b + c}{2}$

Let $a = 4$, $b = x$, then $c = 3x$ so $s = 2 + 2x$, hence

$$A = \sqrt{(2 + 2x)(2x - 2)(2 + x)(2 - x)} = 2\sqrt{(x^2 - 1)(4 - x^2)}$$

2

A is maximum when $x^2 = \frac{5}{2}$, largest area = 3.

2

SOALAN B3

BM Biar f, g dua fungsi yang tertakrif atas $[0, 2c]$ dengan $c > 0$. Tunjukkan bahawa wujud $x, y \in [0, 2c]$ supaya

$$|xy - f(x) + g(y)| \geq c^2.$$

BI Let f, g be two functions defined on $[0, 2c]$ where $c > 0$. Show that there exists $x, y \in [0, 2c]$ such that

$$|xy - f(x) + g(y)| \geq c^2.$$

PENYELESAIAN SOALAN B3

Let $h(x, y) = xy - f(x) + g(y)$. Suppose that $|h(x, y)| < c^2$ for all $0 \leq x, y \leq 2c$.

Then

$$|h(x_1, y_1)| + |h(x_2, y_2)| + |h(x_3, y_3)| + |h(x_4, y_4)| < 4c^2$$

2

for all $0 \leq x_i, y_i \leq 2c$ ($i = 1, 2, 3, 4$).

However, by the triangle inequality, we have

$$\begin{aligned} & |h(0,0)| + |h(0,2c)| + |h(2c,0)| + |h(2c,2c)| \\ & \geq |h(0,0) - h(0,2c) - h(2c,0) + h(2c,2c)| \\ & = 4c^2 \end{aligned}$$

2

which is a **contradiction**.

1

Hence there exists $x, y \in [0, 2c]$ such that

$$|xy - f(x) + g(y)| \geq c^2.$$

1

Note: Jika jawapan shj tanpa jalan kerja beri 2 markah shj.

**SKEMA PENYELESAIAN
BAHAGIAN A
(12 Markah)**

SOALAN A1

BM Misalkan $a_1 = 6, \dots, a_n = 6^{a_n-1}$. Cari baki apabila a_{100} dibahagi dengan 11.

BI Let $a_1 = 6, \dots, a_n = 6^{a_n-1}$. Find the remainder when a_{100} is divided by 11.

PENYELESAIAN SOALAN A1

Fermat's Little Theorem states that if p is prime and p is not divisible by a , then $a^{p-1} \equiv 1 \pmod{p}$. Since 11 is prime and not divisible by 6, then $6^{10} \equiv 1 \pmod{11}$ and $6^n \equiv 6 \pmod{11}$ for all positive n ie $6^n = 6 + 10t$ for some t . Thus

$$a_{100} \equiv 6^{a_{99}} \equiv 6^{6+10t} \equiv 6^6 (6^{10})^t \equiv 6 \pmod{11}$$

Hence the remainder is 6.

Jawapan:	5
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SOALAN A2

BM Misalkan $-1 < y < 0 < x < 1$. Jika $A = x^2y$, $B = \frac{1}{x^2y}$, $C = y^2x$ dan $D = \frac{1}{xy^2}$, susunkan daripada nilai yang terkecil kepada yang terbesar.

BI Let $-1 < y < 0 < x < 1$. If $A = x^2y$, $B = \frac{1}{x^2y}$, $C = y^2x$ and $D = \frac{1}{xy^2}$, arrange them from the smallest to the greatest value

PENYELESAIAN SOALAN A2

$$-1 < y < 0 < x < 1 \Rightarrow -1 < x^2y < 0$$

$$-1 < x^2y < 0 \Rightarrow \frac{1}{x^2y} < -1$$

$$-1 < y < 0 < x < 1 \Rightarrow 0 < xy^2 < 1$$

$$0 < xy^2 < 1 \Rightarrow \frac{1}{xy^2} > 1$$

$$\therefore \frac{1}{x^2y} < -1 < x^2y < 0 < xy^2 < 1 < \frac{1}{xy^2}$$

Therefore the arrangement are: **BACD**

Jawapan:	BACD
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SOALAN A3

BM Satu sisi sebuah segitiga berukuran 4 cm. Dua sisi yang lain berukuran dalam nisbah 1:3. Cari luas yang terbesar untuk segitiga ini.

BI *One side of a triangle is 4 cm. The other two sides are in the ratio 1:3. Find the largest area of the triangle.*

PENYELESAIAN SOALAN A3

With Heron's formula the area of the triangle with sides a, b, c is

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Where } s = \frac{a+b+c}{2}$$

Let a = 4, b = x, then c = 3x so s = 2 + 2x, hence

$$A = \sqrt{(2+2x)(2x-2)(2+x)(2-x)} = 2\sqrt{(x^2-1)(4-x^2)}$$

A is maximum when $x^2 = \frac{5}{2}$, largest area = 3.

Jawapan:	3
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SOALAN A4

BM Permudahkan $\log_2 4 \cdot \log_4 6 \cdot \log_6 8 \dots \log_{2n} (2n+2)$.

BI *Simplify $\log_2 4 \cdot \log_4 6 \cdot \log_6 8 \dots \log_{2n} (2n+2)$*

PENYELESAIAN SOALAN A4

$$\log_2 4 \cdot \log_4 6 \cdot \log_6 8 \cdot \dots \cdot \log_{2n} (2n+2) = \log_2 4 \cdot \frac{\log_2 6}{\log_2 4} \cdot \frac{\log_2 8}{\log_2 6} \cdot \dots \cdot \frac{\log_2 2n}{\log_2 (2n-2)} \cdot \frac{\log_2 (2n+2)}{\log_2 2n}$$

$$= \log_2 (2n+2) \text{ atau } \log_2 2(n+1) \text{ atau } 1 + \log_2 (n+1)$$

Jawapan:	$\log_2 (2n+2)$ atau $\log_2 2(n+1)$ atau $1 + \log_2 (n+1)$
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SOALAN A5

BM Tuliskan 580 sebagai hasil tambah dua nombor kuasa dua.

BI Write 580 as a sum of two squares.

PENYELESAIAN SOALAN A5

By prime power the composition, we have

$$\begin{aligned}
 580 &= 2^2 \cdot 5 \cdot 29 \\
 &= 2^2 \cdot (2^2 + 1) \cdot (5^2 + 2^2) \\
 &= 2^2 \cdot ((2.5 + 1.2)^2 + (2.2 - 1.5)^2) \\
 &= 2^2 (12^2 + 1^2) \\
 &= 24^2 + 2^2
 \end{aligned}$$

Jawapan:	$24^2 + 2^2$
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SOALAN A6

BM Misalkan f suatu fungsi tertakrif pada set integer bukan negatif yang memenuhi

$$f(2n+1) = f(n), f(2n) = 1 - f(n).$$

Cari $f(2007)$.

BI Let f be a function defined on non-negative integers satisfying the following conditions

$$f(2n+1) = f(n), f(2n) = 1 - f(n).$$

Find $f(2007)$.

PENYELESAIAN SOALAN A6

$$f(0) = 1/2; f(1) = f(0) = 1/2; \text{ and } f(2) = 1 - f(1) = 1/2;$$

By induction $f(n) = 1/2$; for any n

$$\therefore f(2007) = 1/2$$

Jawapan:	$\frac{1}{2}$
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BAHAGIAN B
(18 Markah)

SOALAN B1

BM Biar f, g dua fungsi yang tertakrif atas $[0, 2c]$ dengan $c > 0$. Tunjukkan bahawa wujud $x, y \in [0, 2c]$ supaya

$$|xy - f(x) + g(y)| \geq c^2.$$

BI Let f, g be two functions defined on $[0, 2c]$ where $c > 0$. Show that there exists $x, y \in [0, 2c]$ such that

$$|xy - f(x) + g(y)| \geq c^2.$$

PENYELESAIAN SOALAN B1

Let $h(x, y) = xy - f(x) + g(y)$. Suppose that $|h(x, y)| < c^2$ for all $0 \leq x, y \leq 2c$.

Then

$$|h(x_1, y_1)| + |h(x_2, y_2)| + |h(x_3, y_3)| + |h(x_4, y_4)| < 4c^2$$

2

for all $0 \leq x_i, y_i \leq 2c$ ($i = 1, 2, 3, 4$).

However, by the triangle inequality, we have

$$\begin{aligned} & |h(0,0)| + |h(0,2c)| + |h(2c,0)| + |h(2c,2c)| \\ & \geq |h(0,0) - h(0,2c) - h(2c,0) + h(2c,2c)| \\ & = 4c^2 \end{aligned}$$

2

which is a **contradiction**.

1

Hence there exists $x, y \in [0, 2c]$ such that

$$|xy - f(x) + g(y)| \geq c^2.$$

1

Note: Jika jawapan shj tanpa jalan kerja beri 2 markah shj.

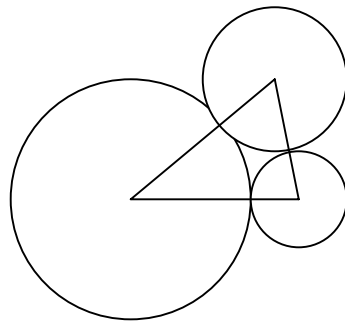
SOALAN B2

BM Dua bulatan masing-masing berjari 1 dan 2 bersentuhan sesama sendiri secara luaran. Suatu bulatan lain dilukis bersentuhan dengan dua bulatan ini dengan pusat-pusat bulatan membentuk suatu segitiga bersudut tepat. Cari jejari bulatan ketiga.

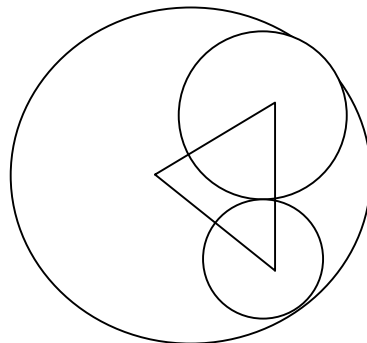
BI *Two circles of radius 1 and 2 respectively are tangential to one another externally. Another circle is drawn tangential to both circles such that their centres form a right angle triangle. Find the radius of the third circle.*

PENYELESAIAN SOALAN B2

Bulatan ketiga boleh bersentuh secara luaran atau kedua-dua bulatan yang diberi terterap dalam bulatan ketiga.



1



Two possibilities:

If drawn externally:

let the radius of third circle be r we have the sides of triangle $r+1$, $r+2$, and 3

1

we will have two possibilities of right angle,

Then first possibility

$$(r + 2)^2 = 3^2 + (r + 1)^2$$

Solving for r we get $r = 3$

2

Next possibility , $(r + 2)^2 + (r + 1)^2 = 3^2$

Solve for r we get $r = \frac{\sqrt{17} - 3}{2}$ 2

If drawn enclosing the two circles, sides of triangle are r-1, r-2 and 3 1

Two possibilities $(r - 1)^2 = 3^2 + (r - 2)^2$ and $(r - 2)^2 + (r - 1)^2 = 3^2$ 2

We get $r = 6$ and

$$r = \frac{\sqrt{17} + 3}{2} \quad \text{2}$$

SOALAN B3

BM Tentukan nilai maksimum bagi $m^2 + n^2$ untuk $m, n \in \{1, 2, 3, \dots, 2007\}$ dan $(n^2 - mn - m^2)^2 = 1$

BI Determine the maximum value of $m^2 + n^2$ where $m, n \in \{1, 2, 3, \dots, 2007\}$ and $(n^2 - mn - m^2)^2 = 1$

PENYELESAIAN SOALAN B3

Let the pair be (m, n) satisfying both conditions. 2

If $m = 1$, then $(1, 1)$ and $(2, 1)$ are the only possibilities. Suppose that (n_1, n_2) is one of the possible solutions with $n_2 > 1$. As $n_1(n_1 - n_2) = n_2^2 \pm 1 > 0$ then we must have $n_1 > n_2$.

Now let $n_3 = n_1 - n_2$. Then

$1 = (n_1^2 - n_1 n_2 - n_2^2)^2 = (n_2^2 - n_2 n_3 - n_3^2)^2$ making (n_2, n_3) as one of possible solutions too with $n_3 > 1$. In the same way we conclude $n_2 > n_3$. The same goes to (n_3, n_4) such that $n_4 = n_2 - n_3$. Hence $n_1 > n_2 > n_3 > \dots$ and must terminate ie when $n_k = 1$ for some k. Since $(n_{k-1}, 1)$ is one of the possibilities, thus we must have n_{k-1} . It looks like the sequence goes 1, 2, 3, 5, 8, ..., 987, 1597 (<2007), a truncated Fibonacci sequence. 3

It is clear that the largest possible pair is $(1597, 987)$ 1