SKEMA PENYELESAIAN
BAHAGIAN A
(12 Markah)

SOALAN A1

BM Seorang lelaki memandu pada kelajuan 90 km/j. Apabila menyedari dia telah terlewat, dia meningkatkan kelajuannya kepada 110 km/j dan melengkapkan perjalanannya sejauh 395 km dalam masa 4 jam. Berapa lamakah dia memandu pada kelajuan 90 km/j?

BI A man was driving at 90km/h. Realizing that he was late, he increased his speed to 110km/h and completed his journey of 395 km in 4 hrs. For how long did he drive at 90 km/h?

PENYELESAIAN SOALAN A1

Let the time he drives at 90 km/hr = t and the time he drives at 110 km/h = 4 – t
So,

$90t + 110(4 - t) = 395$

$20t = 45$

$t = 2.25$ jam atau 2 jam 15 minit atau 135 minit

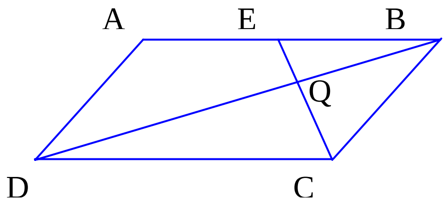
Jawapan:	t = 2.25 jam atau 2 jam 15 minit atau 135 minit

SOALAN A2

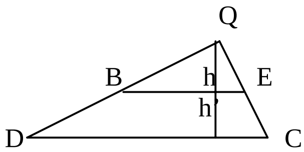
BM Misalkan ABCD satu segiempat selari dengan E satu titik di atas garis AB dengan keadaan 3BE = 2DC. Garis CE dan garis BD bersilang di titik Q. Jika luas ΔDQC ialah 36, cari luas ΔBQE.

BI Let ABCD be a parallelogram where E is a point on AB such that 3BE = 2DC. The lines CE and BD intersect at the point Q. If the area of ΔDQC is 36, find the area of ΔBQE.

PENYELESAIAN SOALAN A2



ΔDQC and ΔBQE are similar



$$\frac{BE}{DC} = \frac{2}{3} \Rightarrow \frac{h}{h'} = \frac{2}{3}$$

Area of ΔDQC = $\frac{1}{2} DC \times h = 36 \Rightarrow DC \times h = 72$

Area of ΔBQE = $\frac{1}{2} BE \times h = \frac{1}{2} \times \frac{2}{3} DC \times \frac{2}{3} h' = \frac{2}{9} DC \times h' = \frac{2}{9} \times 72 = 16$

OR

Since ΔDQC and ΔBQE are similar, and since 3BE = 2DC,
All corresponding sides the same ratio

$$\frac{BE}{DE} = \frac{QE}{QC} = \frac{BQ}{DQ} = \frac{h}{h'} = \frac{2}{3} \quad \therefore \quad \frac{\Delta BQE}{\Delta DQC} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Thus area of ΔBQE = $\frac{4}{9}$ area ΔDQC = 16

Jawapan:	16

SOALAN A3

BM Selesaikan $200520062007^2 - 200520062006 \times 200520062008$.

BI *Solve* $200520062007^2 - 200520062006 \times 200520062008$.

PENYELESAIAN SOALAN A3

Jika $t = 200520062007$ maka

$200520062007^2 - 200520062006 \times 200520062008$ =

$t^2 - (t-1)(t+1) = t^2 - (t^2 - 1) = 1$

Jawapan:	1

SOALAN A4

BM Misalkan $0 < x < 1$. Jika A= x, B= x^2 , C= $\frac{1}{x}$ dan D= $\frac{1}{x^2}$ susunkan daripada nilai yang terkecil kepada yang terbesar.

BI *Let* $0 < x < 1$. *If* $A = x$, $B = x^2$, $C = \frac{1}{x}$ *and* $D = \frac{1}{x^2}$ *arrange them from the smallest to the largest value.*

PENYELESAIAN SOALAN A4

$$0 < x < 1 \Rightarrow x^2 < x$$
$$0 < x < 1 \Rightarrow \frac{1}{x} > 1$$
$$x^2 < x \Rightarrow \frac{1}{x^2} > \frac{1}{x}$$
$$\therefore 0 < x^2 < x < 1 < \frac{1}{x} < \frac{1}{x^2}$$

Therefore the arrangements are: **BACD**

Jawapan:	**BACD**

SOALAN A5

BM Cari integer x, y dan z yang memenuhi $xy + \frac{1}{z} = yz + \frac{1}{x}$ dengan $x \neq z$.

BI *Find integers* x, y *and* z *satisfying* $xy + \frac{1}{z} = yz + \frac{1}{x}$ *where* $x \neq z$.

PENYELESAIAN SOALAN A5

$$xy + \frac{1}{z} = yz + \frac{1}{x} \Rightarrow (x - z)y = \frac{1}{x} - \frac{1}{z} = \frac{z - x}{xz} \Rightarrow xyz = -1.$$

Since x, y, z are integers and $x \neq z$, the only possibilities are $(x, y, z) = (1, 1, -1)$ or $(x, y, z) = (-1, 1, 1)$.

Jawapan:	$(x, y, z) = (1, 1, -1)$ atau $(x, y, z) = (-1, 1, 1)$

SOALAN A6

BM 20 orang menggali sebuah kolam ikan selama 12 hari jika mereka bekerja selama 6 jam sehari. Berapakah bilangan hari yang diperlukan untuk menggali kolam yang sama jika 4 orang bekerja selama 5 jam sehari?

BI 20 persons dig a fish pond in 12 days if they work 6 hours per day. How many days is required to dig the same pond if 4 persons work for 5 hours per day?

PENYELESAIAN SOALAN A6

$20 \times 6 \times 12$ is for one work done

Let x be the number of days required by 4 persons working for 5 hours per day

So, $x = \dfrac{20\times 6\times 12}{4\times 5} = 72$ days

Jawapan:	72 hari

BAHAGIAN B
(18 Markah)

SOALAN B1

BM Dengan menggunakan angka 1, 2, 3, 6, 7, 9, 0 sahaja, tentukan angka mana yang boleh dipadankan dengan huruf di bawah supaya hasil tambahnya adalah betul.

	P	A	K
+	M	A	K
P	I	U	T

BI *Using only the numbers 1, 2, 3, 6, 7, 9, 0, find which number goes with each letter in the addition problem below to make it correct.*

	P	A	K
+	M	A	K
P	I	U	T

PENYELESAIAN SOALAN B1

Hasiltambah tiga digit ≤ 2000 jadi P=1, M=9 I=0.
U dan T tidak boleh 4
K tidak boleh 7 atau 2.
Mak ayang tiggal ialah 3, 6 .
Kalau K=3 maka T=6, dan A=2
Jika K=6 maka A=3, dan T=2

Note: Kaedah cuba jaya tak diterima.

ATAU

$K + K = 2K = T \Rightarrow$ T must be an even number
T = mod 2, 6, 0, but $T \neq$ mod 0 because then K = T
$\therefore$ T = mod 2, 6 $\Rightarrow$ K = 1, 3, 6 [1]
Likewise, 2A = U
If T < 10, then U is even
If T > 10, then U is odd [1]
In both cases, A = 1, 3, 6
Also P + M = I = 10(P) + I < 20
So P< 2, that is, P = 1 [1]

Test for all possibilities:
K, A $\neq$ 1, because P = 1
If K = 3, T =6, then A = 3 or 6 which is not possible
$\therefore$ K = 6, T = 12 mod 10 = 2
A =3, U = 3 + 3 + 1 = 7 because T > 10

Since P = 1, M= 9 and I = 10 mod 10 = 0

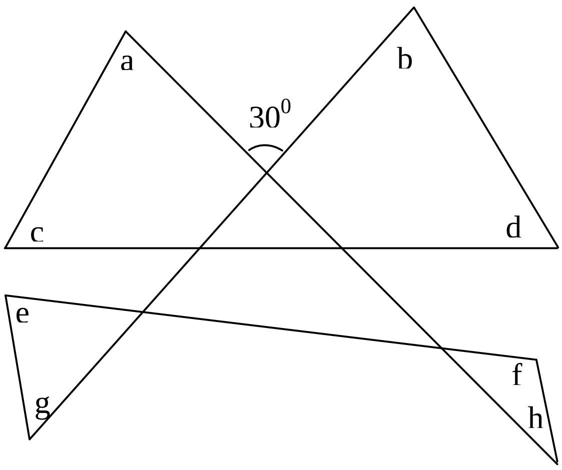
$$\therefore \quad \begin{array}{r} PAK \\ +\ MAK \\ \hline PIUT \end{array} \qquad \begin{array}{r} 1\ 3\ 6 \\ +\ 9\ 3\ 6 \\ \hline 1\ 0\ 7\ 2 \end{array}$$

2

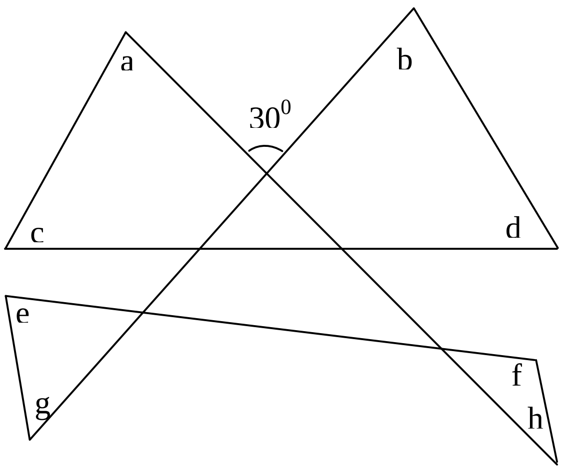
Note: Kaedah cuba jaya tak diterima.

SOALAN B2

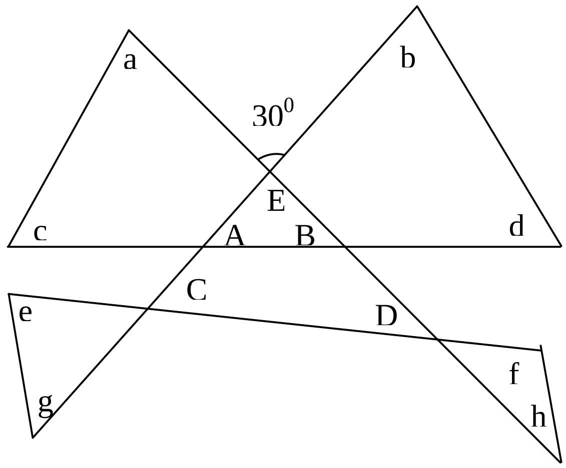
BM Cari jumlah sudut-sudut a+b+c+d+e+f+g+h dalam rajah berikut.



BI Find the sum of angles a+b+c+d+e+f+g+h in the following diagram.



PENYELESAIAN SOALAN B2



It is clear that $A+B+E = \pi$

$C+D+E = \pi$

$B+a+c = \pi$

$A+b+d = \pi$

$C+e+g = \pi$

$D+f+h = \pi$

2

Hence $\{a+b+c+d+e+f+g+h\} + \{A+B+C+D\} = 4\pi$

$\{a+b+c+d+e+f+g+h\} + 2\pi - 2E = 4\pi$.

Since $E = 30^o$, thus $\{a+b+c+d+e+f+g+h\} = 420^o$

4

SOALAN B3

BM Diberi $8\left(y^2+\frac{1}{y^2}\right)-56\left(y+\frac{1}{y}\right)+112=0$ cari semua nilai $y^2+\frac{1}{y^2}$.

BI *Given* $8\left(y^2+\frac{1}{y^2}\right)-56\left(y+\frac{1}{y}\right)+112=0$ *find all the values of* $y^2+\frac{1}{y^2}$.

PENYELESAIAN SOALAN B3

$$8\left(y^2+\frac{1}{y^2}\right)-56\left(y+\frac{1}{y}\right)+112=0$$

Let $u=y+\frac{1}{y}\Rightarrow u^2-2=y^2+\frac{1}{y^2}$. [3]

$$8(u^2-2)-56u+112=0$$

$$8u^2-56u+96=0$$

$$u^2-7u+12=0$$

$$(u-3)(u-4)=0$$ [1]

$$u=3,4$$

$y^2+\frac{1}{y^2}=16-2$ *atau* $9-2$ [2]

$=14$ 7

SKEMA PENYELESAIAN
BAHAGIAN A
(12 Markah)

SOALAN A1

BM Diberi x + y = 2 dan $2xy - z^2 = 1$. Dapatkan penyelesaian integer untuk persamaan-persamaan ini.

BI *Given* $x + y = 2$ *and* $2xy - z^2 = 1$. *Find the integer solutions of the equations.*

PENYELESAIAN SOALAN A1

x+y = 2 and $2xy - z^2 = 1$ leads to $2(x-1)^2 + z^2 = 1$, hence integer solutions are (1,1,1),(1,1,-1)

Jawapan:	(1,1,1),(1,1,-1)

SOALAN A2

BM Dalam suatu kejohanan sukan yang berlangsung selama 4 hari, terdapat n pingat untuk dimenangi. Pada hari pertama, 1/5 daripada n pingat dimenangi. Pada hari kedua, 2/5 daripada baki pingat pada hari pertama dimenangi. Pada hari ketiga, 3/5 daripada baki pingat pada hari kedua dimenangi. Pada hari keempat, 24 pingat dimenangi. Berapakah jumlah pingat kesemuanya?

BI *In a sport's tournament lasting for 4 days, there are* n *medals to be won. On the first day, 1/5 of the* n *medals are won. On the second day, 2/5 of the remainder from the first day are won. On the third day, 3/5 of the remainder from the second day are won. On the final day, 24 medals are won. What was the total number of medals?*

PENYELESAIAN SOALAN A2

Let T_i be the number of medals won on the i th day, $i = 1,2,3,4$.
Then

$$T_1 = \frac{1}{5}n,$$

$$T_2 = \frac{2}{5}(n - T_1) = \frac{8n}{25},$$

$$T_3 = \frac{3}{5}\left(n - \left(T_1 + T_2\right)\right) = \frac{36n}{125}.$$

$$T_4 = 24.$$

$T_1 + T_2 + T_3 + T_4 = n$

$\frac{101n}{125} + 24 = n$

$\frac{24n}{125} = 24$

$\therefore n = 125$

Jawapan:	n=125

SOALAN A3

BM Misalkan **2xyz7** suatu nombor lima angka sedemikian hingga hasil darab angka-angka tersebut ialah sifar dan hasil tambah angka-angka tersebut pula boleh dibahagikan dengan 9. Cari bilangan nombor-nombor tersebut.

BI *Let **2xyz7** be a five-digit number such that the product of the digits is zero and the sum of the digits is divisible by 9. Find how many such numbers.*

PENYELESAIAN SOALAN A3

One of the digits must be zero. The sum of the other two digits must be divisible by 9.
Possible pairs are : (0,0),(0,9),(9,9),(1,8),(2,7),(3,6),(4,5).
The total number of such numbers with the given pair :
(0,0) 1, (0,9) 3, (9,9) 3, (1,8) 6, (2,7) 6, (3,6) 6, (4,5) 6.
There are 31 such numbers

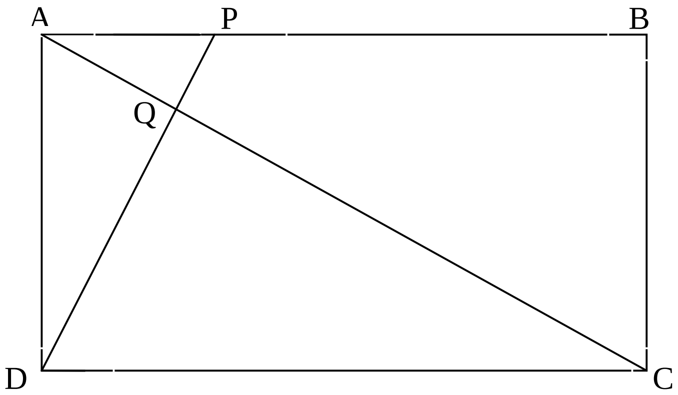
Jawapan:	31

SOALAN A4

BM Misal ABCD sebagai suatu segiempat tepat. Garis DP memotong pepenjuru AC pada Q dan membahagikannya pada nisbah 1:4. Jika luas segitiga APQ satu unit persegi, tentukan luas segiempat tersebut.

BI *Let ABCD be a rectangle. The line DP intersects the diagonal AC at Q and divides it in the ratio of 1:4. If the area of triangle APQ is one unit square, determine the area of the rectangle.*

PENYELESAIAN SOALAN A4



Biar tinggi segitiga APQ ialah x, dan tinggi segitiga AQP ialah y. Segitiga APQ dan segitiga CDQ adalah sebentuk, maka
AP: DC = AQ:QC = x:y = 1: 4. Luas segiempat DC (x+y) = 4AP (x + 4x) = 20 AP.x = 40 (Luas segitiga APQ) = 40

Jawapan:	40

SOALAN A5

BM Cari integer terkecil yang memenuhi syarat apabila dibahagi dengan 2 meninggalkan baki 1, apabila dibahagi dengan 3 meninggalkan baki 2, apabila dibahagi dengan 4 meninggalkan baki 3 dan apabila dibahagi dengan 5 meninggalkan baki 4.

BI *Find the smallest integer such that if divided by 2 leaves a remainder of 1, if divided by 3 leaves a remainder of 2, if divided by 4 leaves a remainder of 3, and if divided by 5 leaves a remainder of 4.*

PENYELESAIAN SOALAN A5

Let N be the integer. Then
$N = 2q_1 + 1$
$= 3q_2 + 2$
$= 4q_3 + 3$
$= 5q_4 + 4$
Observe that
$N + 1 = 2q_1 + 2 = 3q_2 + 3 = 4q_3 + 4 = 5q_4 + 5$
$= 2(q_1 + 1) = 3(q_2 + 1) = 4(q_3 + 1) = 5(q_4 + 1)$

$\therefore$ $2|N + 1$, $3|N+1$, $4|N + 1$ dan $5|N+1$

$\Rightarrow N + 1 = \text{LCM}(2, 3, 4, 5) = 60$

$\therefore N = 59$

Jawapan:	59

SOALAN A6

BM Andaikan *f* suatu fungsi ditakrif pada integer sedemikian

$f(2n) = -2f(n), f(2n+1)= f(n) -1$, dan $f(0) = 2$.

Cari nilai $f(2007)$.

BI *Let f be a function defined on integers such that*

$f(2n) = - 2f(n), f(2n+1) = f(n)-1$, *and* $f(0) = 2$.

Find the value of $f(2007)$.

PENYELESAIAN SOALAN A6

$f(2007) = f(1003) -1 = f(501)-2 = f(250) -3 = -2f(125)-3$

$= -2f(62)-1 = 4f(31) - 2 =4 f(15) - 6 = 4f(7) - 10 =4 f(3)-14$

$= 4f(1) -18 = -14$

Jawapan:	-14

BAHAGIAN B
(18 Markah)

SOALAN B1

BM Diberi $8\left(y^2+\frac{1}{y^2}\right)-56\left(y+\frac{1}{y}\right)+112=0$ cari semua nilai $y^2+\frac{1}{y^2}$.

BI *Given* $8\left(y^2+\frac{1}{y^2}\right)-56\left(y+\frac{1}{y}\right)+112=0$ *find all the values of* $y^2+\frac{1}{y^2}$.

PENYELESAIAN SOALAN B1

$$8\left(y^2+\frac{1}{y^2}\right)-56\left(y+\frac{1}{y}\right)+112=0$$

Let $u=y+\frac{1}{y}\Rightarrow u^2-2=y^2+\frac{1}{y^2}$. [3]

$8(u^2-2)-56u+112=0$

$8u^2-56u+96=0$

$u^2-7u+12=0$

$(u-3)(u-4)=0$ [1]

$u=3,4$

$y^2+\frac{1}{y^2}=16-2$ *atau* $9-2$ [2]

$=14$ $\quad 7$

SOALAN B2

BM Satu sisi sebuah segitiga berukuran 4 cm. Dua sisi yang lain berukuran dalam nisbah 1:3. Cari luas yang terbesar untuk segitiga ini.

BI *One side of a triangle is 4cm. The other two sides are in the ratio 1:3. What is the largest area of the triangle?*

PENYELESAIAN SOALAN B2

With Heron's formula the area of the triangle with sides a, b , c is

A = $\sqrt{s(s-a)(s-b)(s-c)}$ [2]

Where s = $\dfrac{a+b+c}{2}$

Let a = 4, b = x, then c = 3x so s = 2 + 2x, hence
A = $\sqrt{(2+2x)(2x-2)(2+x)(2-x)} = 2\sqrt{(x^2-1)(4-x^2)}$ [2]

A is maximum when $x^2 = \dfrac{5}{2}$, largest area = 3. [2]

SOALAN B3

BM Biar f, g dua fungsi yang tertakrif atas $[0, 2c]$ dengan $c > 0$. Tunjukkan bahawa wujud $x, y \in [0, 2c]$ supaya

$$|xy - f(x) + g(y)| \geq c^2 .$$

BI *Let* f, g *be two functions defined on* $[0, 2c]$ *where* $c > 0$. *Show that there exists* $x, y \in [0, 2c]$ *such that*

$$|xy - f(x) + g(y)| \geq c^2 .$$

PENYELESAIAN SOALAN B3

Let $h(x,y) = xy - f(x) + g(y)$. Suppose that $|h(x,y)| < c^2$ for all $0 \leq x, y \leq 2c$.

Then

$$|h(x_1,y_1)| + |h(x_2,y_2)| + |h(x_3,y_3)| + |h(x_4,y_4)| < 4c^2$$ [2]

for all $0 \leq x_i, y_i \leq 2c$ $(i = 1,2,3,4)$.

However, by the triangle inequality, we have

$$\begin{aligned} & |h(0,0)| + |h(0,2c)| + |h(2c,0)| + |h(2c,2c)| \\ \geq\ & |h(0,0) - h(0,2c) - h(2c,0) + h(2c,2c)| \\ =\ & 4c^2 \end{aligned}$$ [2]

which is a **contradiction**. [1]

Hence there exists $x, y \in [0,2c]$ such that

$$|xy - f(x) + g(y)| \geq c^2 .$$ [1]

Note: Jika jawapan shj tanpa jalan kerja beri 2 markah shj.

SKEMA PENYELESAIAN
BAHAGIAN A
(12 Markah)

SOALAN A1

BM Misalkan $a_1 = 6, ..., a_n = 6^{a_{n-1}}$. Cari baki apabila a_{100} dibahagi dengan 11.

BI *Let* $a_1 = 6, \ldots, a_n = 6^{a_{n-1}}$. *Find the remainder when* a_{100} *is divided by 11.*

PENYELESAIAN SOALAN A1

Fermat's Little Theorem states that if *p* is prime and p is not divisible by *a*, then $a^{p-1} \equiv 1 \bmod p$. Since 11 is prime and not divisible by 6, then $6^{10} \equiv 1 \bmod 11$ and $6^n \equiv 6 \bmod 10$ for all positive n ie $6^n = 6 + 10t$ for some t. Thus

$$a_{100} \equiv 6^{a_{99}} \equiv 6^{6+10t} \equiv 6^6\left(6^{10}\right)^t \equiv 6 \bmod 11$$

Hence the remainder is 6.

Jawapan:	5

SOALAN A2

BM Misalkan $-1 < y < 0 < x < 1$. Jika A = x^2y, B = $\frac{1}{x^2y}$, C = y^2x dan D = $\frac{1}{xy^2}$, susunkan daripada nilai yang terkecil kepada yang terbesar.

BI *Let* $-1 < y < 0 < x < 1$. *If* $A = x^2y$, $B = \frac{1}{x^2y}$, $C = y^2x$ *and* $D = \frac{1}{xy^2}$, *arrange them from the smallest to the greatest value*

PENYELESAIAN SOALAN A2

$$-1 < y < 0 < x < 1 \Rightarrow -1 < x^2y < 0$$

$$-1 < x^2y < 0 \Rightarrow \frac{1}{x^2y} < -1$$

$$-1 < y < 0 < x < 1 \Rightarrow 0 < xy^2 < 1$$

$$0 < xy^2 < 1 \Rightarrow \frac{1}{xy^2} > 1$$

$$\therefore \frac{1}{x^2y} < -1 < x^2y < 0 < xy^2 < 1 < \frac{1}{xy^2}$$

Therefore the arrangement are: **BACD**

Jawapan:	**BACD**

SOALAN A3

BM Satu sisi sebuah segitiga berukuran 4 cm. Dua sisi yang lain berukuran dalam nisbah 1:3. Cari luas yang terbesar untuk segitiga ini.

BI *One side of a triangle is 4 cm. The other two sides are in the ratio 1:3. Find the largest area of the triangle.*

PENYELESAIAN SOALAN A3

With Heron's formula the area of the triangle with sides a, b , c is
A = $\sqrt{s(s-a)(s-b)(s-c)}$
Where s = $\dfrac{a+b+c}{2}$
Let a = 4, b = x, then c = 3x so s = 2 + 2x, hence
A = $\sqrt{(2+2x)(2x-2)(2+x)(2-x)} = 2\sqrt{(x^2-1)(4-x^2)}$
A is maximum when $x^2 = \dfrac{5}{2}$, largest area = 3.

Jawapan:	3

SOALAN A4

BM Permudahkan $\log_2 4 \cdot \log_4 6\ \log_6 8\ ...\ \log_{2n}(2n+2)$.

BI *Simplify* $\log_2 4 \cdot \log_4 6\ \log_6 8\ ...\ \log_{2n}(2n+2)$

PENYELESAIAN SOALAN A4

$$\log_2 4 \cdot \log_4 6 \cdot \log_6 8 \cdot ... \cdot \log_{2n}(2n+2) = \log_2 4 \cdot \frac{\log_2 6}{\log_2 4} \cdot \frac{\log_2 8}{\log_2 6} \cdot ... \cdot \frac{\log_2 2n}{\log_2(2n-2)} \cdot \frac{\log_2(2n+2)}{\log_2 2n}$$

$= \log_2(2n+2)$ *atau* $\log_2 2(n+1)$ *atau* $1+\log_2(n+1)$

Jawapan:	$\log_2(2n+2)$ atau $\log_2 2(n+1)$ atau $1+\log_2(n+1)$

SOALAN A5

BM Tuliskan 580 sebagai hasil tambah dua nombor kuasa dua.

BI *Write 580 as a sum of two squares.*

PENYELESAIAN SOALAN A5

By prime power the composition, we have

$$580 = 2^2.5.29$$
$$= 2^2.(2^2+1).(5^2+2^2)$$
$$= 2^2((2.5+1.2)^2+(2.2-1.5)^2)$$
$$= 2^2(12^2+1^2)$$
$$= 24^2+2^2$$

Jawapan:	$24^2 + 2^2$

SOALAN A6

BM Misalkan *f* suatu fungsi tertakrif pada set integer bukan negatif yang memenuhi

$$f(2n+1) = f(n),\ f(2n) = 1 - f(n).$$

Cari *f(2007).*

BI *Let f be a function defined on non-negative integers satisfying the following conditions*

$$f(2n+1) = f(n),\ f(2n) = 1 - f(n).$$

Find f(2007).

PENYELESAIAN SOALAN A6

$f(0) = ½; f(1) = f(0) = ½;$ and $f(2) = 1 - f(1) = ½;$
By induction $f(n) = ½;$ for any n
$\therefore f(2007) = ½$

Jawapan:	$\frac{1}{2}$

BAHAGIAN B
(18 Markah)

SOALAN B1

BM Biar f, g dua fungsi yang tertakrif atas $[0, 2c]$ dengan $c > 0$. Tunjukkan bahawa wujud $x, y \in [0, 2c]$ supaya

$$|xy - f(x) + g(y)| \geq c^2 .$$

BI *Let* f, g *be two functions defined on* $[0, 2c]$ *where* $c > 0$. *Show that there exists* $x, y \in [0, 2c]$ *such that*

$$|xy - f(x) + g(y)| \geq c^2 .$$

PENYELESAIAN SOALAN B1

Let $h(x, y) = xy - f(x) + g(y)$. Suppose that $|h(x, y)| < c^2$ for all $0 \leq x, y \leq 2c$.

Then

$$|h(x_1, y_1)| + |h(x_2, y_2)| + |h(x_3, y_3)| + |h(x_4, y_4)| < 4c^2$$ [2]

for all $0 \leq x_i, y_i \leq 2c$ $(i = 1, 2, 3, 4)$.

However, by the triangle inequality, we have

$$\begin{aligned} & |h(0,0)| + |h(0,2c)| + |h(2c,0)| + |h(2c,2c)| \\ \geq\ & |h(0,0) - h(0,2c) - h(2c,0) + h(2c,2c)| \\ =\ & 4c^2 \end{aligned}$$ [2]

which is a **contradiction**. [1]

Hence there exists $x, y \in [0, 2c]$ such that

$$|xy - f(x) + g(y)| \geq c^2 .$$ [1]

Note: Jika jawapan shj tanpa jalan kerja beri 2 markah shj.

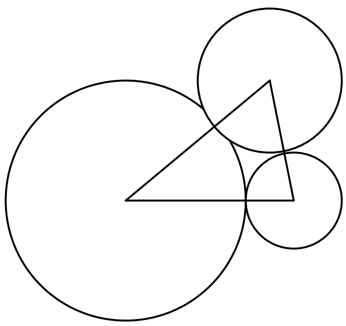
SOALAN B2

BM Dua bulatan masing-masing berjejari 1 dan 2 bersentuhan sesama sendiri secara luaran. Suatu bulatan lain dilukis bersentuhan dengan dua bulatan ini dengan pusat-pusat bulatan membentuk suatu segitiga bersudut tepat. Cari jejari bulatan ketiga.

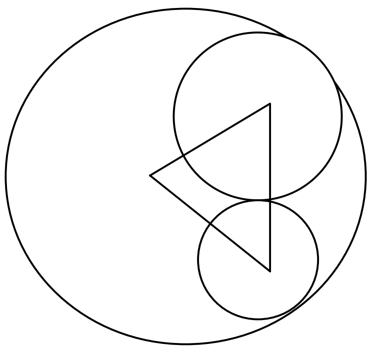
BI *Two circles of radius 1 and 2 respectively are tangential to one another externally. Another circle is drawn tangential to both circles such that their centres form a right angle triangle. Find the radius of the third circle.*

PENYELESAIAN SOALAN B2

Bulatan ketiga boleh bersentuh secara luaran atau kedua-dua bulatan yang diberi terterap dalam bulatan ketiga.



1



Two possiblities:
If drawn externally:
let the radius of third circle be r we have the sides of triangle r+1, r+2, and 3 1

we will have two possibilities of right angle,
Then first possiblitiy

$(r+2)^2 = 3^2 + (r+1)^2$

Solving for r we get r = 3 2

Next possibility , $(r+2)^2 + (r+1)^2 = 3^2$

Solve for r we get $r = \dfrac{\sqrt{17} - 3}{2}$ [2]

If drawn enclosing the two circles, sides of triangle are
r-1, r-2 and 3 [1]

Two possiblities $(r-1)^2 = 3^2 + (r-2)^2$ and $(r-2)^2 + (r-1)^2 = 3^2$ [2]

We get r = 6 and

$$r = \frac{\sqrt{17} + 3}{2}$$ [2]

SOALAN B3

BM Tentukan nilai maksimum bagi $m^2 + n^2$ untuk $m,n \in \{1,2,3,\ldots,2007\}$ dan $(n^2 - mn - m^2)^2 = 1$

BI *Determine the maximum value of* $m^2 + n^2$ *where* $m,n \in \{1,2,3,\ldots,2007\}$ *and* $(n^2 - mn - m^2)^2 = 1$

PENYELESAIAN SOALAN B3

Let the pair be (m,n) satisfying both conditions. [2]

If m = 1, then $(1,1)$ and $(2,1)$ are the only possibilities. Suppose that (n_1, n_2) is one of the possible solutions with $n_2 > 1$. As $n_1(n_1 - n_2) = n_2^2 \pm 1 > 0$ then we must have $n_1 > n_2$.

Now let $n_3 = n_1 - n_2$. Then

$1 = (n_1^2 - n_1 n_2 - n_2^2)^2 = (n_2^2 - n_2 n_3 - n_3^2)^2$ making (n_2, n_3) as one of possible solutions too with $n_3 > 1$. In the same way we conclude $n_2 > n_3$. The same goes to (n_3, n_4) such that $n_4 = n_2 - n_3$. Hence $n_1 > n_2 > n_3 > \ldots$ and must terminate ie when $n_k = 1$ for some k. Since $(n_{k-1}, 1)$ is one of the possibilities, thus we must have n_{k-1}. It looks like the sequence goes 1,2,3,5,8, …, 987, 1597 (<2007), a truncated Fibonacci sequence. [3]

It is clear that the largest possible pair is (1597, 987) [1]